Optimal Control of Unreliable Production/Inventory Systems with Expediting

Yan Jia and Ratna Babu Chinnam
Department of Industrial & Manufacturing Engineering
Wayne State University

In this paper, we study an unreliable multi-product production/inventory system consisting of $N$ production machines and an assembly machine. The final products are assembled-to-order whereas components are built at $N$ separate production machines on a make-to-stock routine. The objective of our study is to develop an integrated optimal control policy that minimizes the total long-run average cost of production, expediting, inventory, preventive maintenance and repair. We present a three-critical-limit policy that addresses the production/expediting/inventory and preventive maintenance controls in an integrated manner. Furthermore, we propose a simulation based experimental design scheme based on response surface methodology to optimize the parameters of the policy. Finally, we provide an illustrative numerical example.

Keywords: Assemble-to-order, expediting, preventive maintenance

1. Introduction

With ever increasing competition in the currently global marketplace, more and more companies are turning to the assemble-to-order (ATO) strategy to stay responsive and cost efficient within the supply chain. This strategy enables companies to pool component inventories and postpone the final assembly until the demand is realized, greatly shortening response time to customers, and allowing higher product variety. Component supply lead-times are however often long and the supply process are unreliable and/or capacitated. In these cases, companies need to incorporate expediting techniques to get the system back in control, and thus, take advantage of the transient market opportunities.

The literature on ATO systems can be broadly classified into two categories: systems with periodic review and systems with continuous review (see (Song et al.2002)) for a comprehensive review). ATO systems with continuous review can be further classified into pure inventory systems (inventory replenishment leadtimes are assumed to be independent of component supply process) and integrated production-inventory systems (inventory replenishment leadtimes are affected by component supply process). For both cases, component inventory is typically assumed to be managed using an independent base-stock policy with a stationary base-stock level. Extant literature mostly offers approximation and heuristic methods to analyze and control ATO systems; the optimal control policy however remains unknown (Song et al.2002).

ATO systems with backordered demand have been studied intensively, but few papers have been written on ATO systems with expediting to meet target leadtimes for customers. This might
be due to the notorious “intractable nature of dual-source models” (Bradley 2004). In the literature, there are two types of expediting: overtime and subcontracting. Overtime refers to production that takes place out of regular-time in-house at a discrete-time interval. Subcontracting means procurement from outside suppliers (Bradley 1997). It can occur in both continuous-time and discrete-time contexts. A number of authors have provided models for overtime production. Dellaert et al. consider lot sizing in ATO systems with overtime and due dates (Dellaert et al.1998). Duenyas investigate the decision of setting the production quota for discrete-time models with and without backlogging (Duenyas et al.1997). Less work appears to have been done on subcontracting. Bradley has considered subcontracting in a make-to-stock (MTS) model with backorders in discrete-time. A stationary two-parameter base-stock policy is shown to be optimal for the infinite horizon discounted case and for the infinite horizon average-cost case where shortfall is bounded (Bradley 1998). Close to our work, Bradley develops an M/M/1 model of subcontracting where MTS production is allowed. This work is extended by Bradley (Bradley 2004) where the relationship between capacity and subcontracting is explicitly considered. In our model, we consider expediting in the form of subcontracting and decide when and how many to expedite in ATO systems with backordered demand.

Queuing systems have long been used in modeling complex manufacturing systems and computer systems (Kleinrock1976, Buzacott et al. 1993, and Song et al.2002). Overtime production in queuing networks has been considered by Karmarkar (Karmarkar et al.1987) and Bitran (Bitran et al. 1991). Extant literature on early queuing models mostly assumes that servers are perfectly reliable, an unrealistic assumption for the real world. Unpredictable machine failures have been identified as the main cause of uncertainty in manufacturing systems. Accordingly, growing interest can be noticed in the literature concerning the modeling of failure-prone production systems and the analysis of the impact of disruptions on system performance. For example, Federgruen (Federgruen et al. 1990) considers an M/G/1 queuing system with an unreliable server under corrective maintenance (CM). However, the importance of preventive maintenance (PM) to reduce the occurrence of machine failures and increase machines' utility rate have been ignored, and optimal PM policies have not been addressed sufficiently. Most papers we reviewed which make an effort to obtain optimal PM policies have focused on conventional maintenance theory which studies isolated machines only by their “technical state” information. Given that each machine is only a part of the production-inventory system, the entire “operational state” information of the system, such as the buffer level of components, has a significant effect on the optimal preventive policy. Therefore, it would be more reasonable to consider the preventive maintenance/failure in the context of an integrated production-inventory system. The work by Das (Das et al.1999) and Iravani (Iravani et al.2002) motivates us for this research. However, their problem settings are quite different. Iravani et al.2002 considers a MTS production/inventory system consisting of a single deteriorating machine which produces a single item. Das et al.1999 consider a production-inventory system, where inventory is maintained according to an \((s, S)\) policy, and the production process is subject to failure. A preventive maintenance policy has been studied, and they employ a simulation-based optimization algorithm (i.e., reinforcement learning) to obtain optimal policy parameters (Das et al. 1999).

In this paper, we consider an unreliable production/inventory system consisting of \(N\) production machines and an assembly machine. Multiple products are ATO whereas the required components are built at \(N\) separate machines on a MTS routine. Once demand for product is
realized, if there is enough deliverable inventory for all components, the demand is filled immediately; otherwise, the demand is backordered.

Based on the earlier work, we formulate the jointly optimal control problem of: (1) deciding the production/expediting/inventory policy of all \( N \) components and (2) designing a preventive maintenance policy for all \( N \) machines, as a semi-Markov decision problem. More specifically, we consider a component supply process with expediting.

Within the context of ATO, there is very little literature on expediting. One of the exceptions is Plambeck (Plambeck et al. 2003), who proves that multi-dimensional ATO control problems can be separated into single-item inventory control problems, and the optimal production and expediting policy for each component is independent of all other components under the assumption that expedited components have zero leadtime and reliable normal production facilities. However, our model assumes unreliable production machines. We adopt quite different production/expediting/inventory policies whereas our preventive maintenance policy is similar to the one of Das et al. 1999. They assume that production/inventory policy follows the base-stock policy \((s, S)\), with the value of \( s \) and \( S \) fixed in the model of Das et al.1999. The most notable characteristic of the control approach proposed in our paper is to consider the integrated effects of preventive maintenance policy and production/expediting/inventory policies on the optimal control of the system.

The remainder of this paper is organized as follows. In Section 2 and Section 3, we present detailed system descriptions and state all the assumptions that our system adheres to. Using our assumptions, we model the optimal control of the unreliable production/inventory system as a semi-Markov decision problem. In Section 4, we present a three-critical-limit policy that addresses the production/expediting/inventory and preventive maintenance controls in an integrated manner. In Section 5, we analyze mathematically the complex probabilistic system to understand the inner working of systems, and thus, obtain useful insights for effective system design and control. In Section 6, we propose a control approach and develop an experimental framework to identify the optimal parameter values. Then, we present numerical results. In the last section, we summarize our research findings and offer directions for future research.

2. System Description

We consider an ATO based unreliable production/inventory system consisting of \( N \) production machines and an assembly machine, as shown in Figure 1. The assumptions and model parameters are described below.

![Figure 1 An ATO based unreliable production inventory system](image)
2.1 Demand Arrival Process

In this paper, orders for each product $i$, $(i=1,2,\ldots,M)$, arrive according to a stochastic Poisson process $D_i(t)$ with rate $\lambda_i$, where $D_i(t)$ denotes the cumulative number of orders by time $t$. Fulfilling an order for product $i$ requires $o_{ij}$ components of type $j$, for $i=1,2,\ldots,M; j=1,2,\ldots,N$. In addition, each order for product $i$ must be fulfilled within the target leadtime $L_i$ (constant for any given product but varies across products).

2.2 Product Assembly Process

An order for product $i$ arriving at time $t$ must be assembled by time $t+L_i$ based on a first-in-first-out (FIFO) basis. $A_i(t)$ denotes the number of product $i$ assembled up to time $t$. We assume that the time to assemble the components into the final product is negligible. In order to fulfill a customer order, all components required must be available; otherwise, the order is backordered. In the event one or more components are unavailable upon the demand arrival, the demands for the components are backlogged in respective queues, each with an infinite capacity until all the required components become available.

2.3 Component Supply Process

Replenishment orders for component $j$ are sent to a single unreliable machine $j$, for $j=1,2,\ldots,N$, on which they are processed on a first-come-first-serve (FCFS) basis. The production machine $j$ is subject to variation from preventive and corrective maintenance outages. The natural process time at machine $j$ is assumed a Gamma distribution (Mahadevan 1997). Thus, the supply system can be viewed as $N$ parallel stochastic production machines. $P_j(t)$ denotes the cumulative number of components produced up to time $t$ since the last preventive maintenance or repair completion epoch. The production process of machine $j$ is suspended when either a breakdown occurs or the buffer inventory of component $j$ reaches $S_j$.

We model the problem of finding the optimal expediting policy as a semi-Markov decision process (SMDP). The decision maker observes the system at decision epochs. Depending on the inventory level of components and the status of the production facilities, one decides whether or not to use expediting. If expediting is used, one also ought to determine how much to expedite. $E_j(t)$ denotes the cumulative number of $j$ type components expedited up to time $t$.

2.4 Machine Maintenance or Repair Process

We assume that the production machines are subject to random failures but the assembly machine is not. Simon justifies the assumption in several cases (Simon et al. 1995). Furthermore, we assume that all these $N$ production machines are subject to time-dependent failure, which can occur at any time even when the machines are in a nonoperating state. Compared with operational-dependent failure, which can only occur in an operation, algorithms based on time-based failure will be more easily implementable (Iravani et al. 2000). For each machine $j, j=1, 2,\ldots, N$, we assume the following: 1) hazard rate function is strictly increasing to infinity, 2) expected cost of repair is greater than that of preventive maintenance, and 3) machine will be “as-good-as-new” after preventive maintenance or repair.
3. System Formulation

The objective of our study is to find an optimal integrated control policy that minimizes the long-run expected cost. In this section, we first model the underlying stochastic process as an SMDP by identifying decision epochs, state variables, and the possible action set of each state.

3.1 Decision Epochs

We simulate the system as a discrete-event system where the significant events are: demand arrival, production completion, expediting completion, failure, maintenance completion, and repair completion. In case ties occur, the next event for components or machines with the smallest index occurs first, and so on. If decisions are only restricted to system state change epochs, the decision epochs are demand-arrival epochs, production-completion epochs, expediting-completion epochs, maintenance completion epochs, and repair completion epochs.

3.2 Action

Let $A$ denote the action space for the control actions for each machine and each component. At each decision epoch, for each state, the admissible action could be to Produce, Expedite, perform Preventive Maintenance, and Repair.

3.3 System State

The state of the system at any decision epoch is represented by a vector: $S = \{ (I_j(t), N_j(t)) : j = 1, 2, ..., N; t \geq 0 \}$.

--- $I_j(t)$ denotes the inventory position of component $j$ at time $t$. 

$$I_j(t) = P_j(t) + E_j(t) - \sum_{i=1}^{M} \alpha_{ij} A_i(t)$$

Moreover,

--- $T_j(t)$ denotes the age of machine $j$ since the last maintenance/repair cycle.

3.4 States-Action Space

The complete state-action space over which a learning agent searches for an optimal policy is $E = S \times A$. A vector of size $|S|$ containing actions for each element of the state space is called a control policy. Let $\lambda$ denote the set of feasible control policies.

4. Integrated Control Policy

In this section, we propose a three critical-limit policy that addresses the production/expediting/inventory and preventive maintenance controls in an integrated manner. The first step is to characterize the optimal policy for assembly and expediting.

Under the assumption that expedited components have zero leadtime and that production facilities are perfectly reliable, Plambeck (Plambeck et al. 2003) proves that the multi-dimensional ATO control problem separates into single-item inventory control problems. The
optimal production and expediting policy for each component is independent of all other components. It can also be shown that the above separation principle can be applied within the context of unreliable production machines, as long as the expedited components have zero leadtime.

**Proposition 1.** Under the assumption that order fill rate is 1 and assembly sequencing follows FIFO:

The assembly quantity of product \( i \) up to time \( t \) is:

\[
A_i(t) = D_i(t - L_i), \text{ for } t \geq 0 \text{ and } i = 1, 2, \ldots, M
\]

Inventory position \( I_j(t) \) at time \( t \) is:

\[
I_j(t) = P_j(t') + E_j(t') - \sum_{i=1}^{M} a_{ij} A_i(t'), \text{ for } 0 \leq t' \leq t \text{ and } j = 1, 2, \ldots, N
\]

Production quantity \( P_j(t) \) at time \( t \) is:

\[
P_j(t) = \sup [S - I_j(t)]^+
\]

Expediting quantity \( E_j(t) \) is:

\[
E_j(t) = \sup [\sum_{i=1}^{M} a_{ij} D_i(t' - L_i) - P_j(t') - s_i]_+, \text{ for } 0 \leq t' \leq t \text{ and } j = 1, 2, \ldots, N
\]

Based on the separation principle, we propose that production/expediting/inventory of each component is controlled by separate three critical-limiting policies \( (s_j, S_j, T_j) \). The policy works as follows: At each demand arrival epoch, if the inventory position of each component \( j \), \( I_j(t) < s_j \), then expedite component \( j \). If \( s_j \leq I_j(t) \leq S_j \), produce component \( j \) and do not expedite. If \( I_j(t) > S_j \) and \( P_j(t) > T_j \), preventive maintenance will be conducted on machine \( j \).

5. Mathematical Model

Dynamic control of an ATO system is already challenging and intractable due to the large state space and the underlying stochastic model (Song et al. 2003). The optimal control problem would be more complicated when incorporating PM under the context of an unreliable production-inventory system. As a result, the structure of mathematical models for these stochastic systems is often highly complex, and it is extremely difficult to get an exact, analytic solution. To deal with the difficulty, several earlier works have suggested simulation as an efficient alternative (McCall 1965, Kenne et al. 1997, Gharbi et al. 2000).

In view of this, we propose an alternative control approach that combines the mathematical model with a simulation tool to find optimal parameter values of the proposed control policy. First, we analyze mathematically the complex probabilistic system to understand the inner working of systems and thus obtain useful insights for effective system design and control. Then, we utilize the simulation approach to find the optimal solution that minimizes the steady state average cost. The work similar to ours is by Kenne (Kenne et al. 1997) and Gharbi (Gharbi et al. 2000). They use the combination of both analytical and simulation approaches to analyze a single machine production-inventory system that produces a single component, and the production and demand rates are assumed known and constant. The experimental design framework adopted here to determine the optimal policy however is quite different.

The cost structure consists of preventive maintenance cost, repair cost, and operation cost (holding cost, production cost and expediting cost). Given the requirement that all accepted orders must be filled within the required leadtime (fill rate is 1), the backlogged cost of orders
can be explicitly considered in the expediting cost of required components. The cost incurred between decision epochs depends on the action taken. For the remainder of the paper, we drop the subscript $j$ for notational convenience.

**Notation:**

$p$: the average throughput of machine when it is in operational state

$X$: the lifetime of machine with pdf $f(x)$ and cdf $F(x)$

$H(x)$: the hazard function of $X$, and $H(x) = \frac{f(x)}{F(x)}$

$m$: the mean duration of preventive maintenance on machine

$r$: the mean duration of repair on machine $j$

$c_m$: the unit time cost of preventive maintenance on machine

$c_r$: the unit time cost of repair on machine $j$

$A(t; T)$: the availability of machine under policy $T$, which is given in terms of Mean Time to Failure (MTTF, denoted by $m_f$) and Mean Time to Repair (MTTR, denoted by $m_r$). That is:

$$A(t; T) = \frac{m_f(T)}{m_f(T) + m_r(T)} = \frac{T \bar{F}(T) + E[X|X < T]F(T)}{(T + m)\bar{F}(T) + (E[X|X < T] + r)F(T)}$$

(5)

The throughput of machine under preventive maintenance policy $T$ is given by:

$$P(t; T) = pA(T) = p\frac{T \bar{F}(T) + \int_t^T tf(t)dt}{(T + m)\bar{F}(T) + \int_0^T tf(t)dt + rF(T)}$$

(6)

The inventory position of component under preventive maintenance policy $T$ is given by:

$$I(t; T) = P(t; T) + E(t; T) - \sum_{i=1}^{M} a_i D_i(t; T) - s(T)$$

(7)

The maintenance and repair cost per unit time of machine under preventive maintenance policy $T$ is given by:
For convenience, we first consider minimizing the expected infinite horizon discounted cost of all components production/expediting and all machines maintenance/repair, subject to filling orders for product \( i \) within the target leadtime \( L_i \). Based on the separation principle, the objective is transformed into minimizing the expected infinite horizon discounted production/expediting cost of each component and discounted maintenance/repair cost of its corresponding production machine, so we have the following mathematical program:

\[
M(t; T) = \frac{c_m \overline{F}(T) + c_r F(T)}{(T + m) F(T) + \int_0^T t f(t) dt + r F(T)}
\]  

(8)

**Proposition 2.** If \( \frac{r}{m} + \frac{1}{m f(0)} > \frac{c_r}{c_m} > \frac{r + E[X]}{m + E[X]} \), then \( M(t; T) \) is strongly quasiconvex and there exists a unique \( T^* \equiv \arg\min \{M(t; T)\} \) that satisfies \( 0 < T^* < \infty \).

For convenience, we first consider minimizing the expected infinite horizon discounted cost of all components production/expediting and all machines maintenance/repair, subject to filling orders for product \( i \) within the target leadtime \( L_i \). Based on the separation principle, the objective is transformed into minimizing the expected infinite horizon discounted production/expediting cost of each component and discounted maintenance/repair cost of its corresponding production machine, so we have the following mathematical program:

\[
J(s, S, T) = \min_{(s, S, T)} \mathbb{E}\left\{ \int_0^\infty \exp(-\alpha t) \left[ c_p dP(t; T) + c_v dE^*(t; T) + c_m dI^+(t; T) + dM(t; T) \right] \right\}
\]

\( S.T. \)

\[
E^*(t; T) = \sup_{i=1}^M \left[ a_i D_i(t' - L_i) - P(t'; T) - s^*(T) \right]^+, \text{ for } 0 \leq t' \leq t
\]

(9)

Let

\[
J'(s, S, T) = \min_{(s, S, T)} \mathbb{E}\left\{ \int_0^\infty \exp(-\alpha t) \left[ c_p dP(t; T) + c_v dE^*(t; T) + c_m dI^+(t; T) \right] \right\}
\]

Boukas proved \( J' \) is strictly convex and has a unique minimum point, and it satisfies the well-known Hamilton-Jacobi-Bellman (HJB) equations (Boukas et al. 1990). Based on the analysis and Proposition 2, it is quite clear that the value function \( J(s, S, T) \) is strictly convex and has a unique minimum point \( (s^*, S^*, T^*) \). The long-run expected average cost of interest can be easily obtained by the following transformation.

Let \( \Phi'(s, S, T) \) be its expected infinite horizon total discounted-cost.

\( \Phi'(s, S, T) \) be its long-run average cost.

\( \Phi'(s, S, T) \) be its expected cost up to time \( T \).

Given that the Markov chain is a unichain (i.e., the Markov chain corresponding to every stationary policy has a single recurrent class and possibly empty set of transient states) exists as an immediate consequence of Proposition 11.4.1 and Proposition 11.4.7 of Puterman (Puterman 1994). Therefore, the expected infinite horizon average production/expediting/holding cost of each component and average maintenance/repair cost of its corresponding production machine has the following mathematical structure:
\[ \psi(s,S,T) = \lim_{T \to \infty} \frac{1}{T} \Phi_T(s,S,T) \]
\[ = \lim_{T \to \infty} \frac{1}{T} \int_0^T \exp(-\alpha t)[c_p dP(t,T) + c_d dE(t,T) + c_o dI^+(t,T) + dM(t,T)] \] (10)

Since the HJB equations or the optimality conditions of the control problem mentioned above are complex, it is extremely difficult to obtain analytically the optimal control policy by solving the HJB equations (Refer to (Hu et al. 1995)). In the following section, we adopt a simulation based response surface experimental framework that approximates the relationship between the incurred cost and control variables \(s\), \(S\), and \(T\). The optimal parameter values \((s^*, S^*, T^*)\) for the integrated production/expediting/inventory policies can be obtained from the estimated function.

6. Experimental Designs and Analysis

The best values of the control variables \(s\), \(S\), and \(T\), defined as parameters of a production/expediting/inventory policy, are given by minimizing the estimated relationship between cost and control variables. Thus, the aim of the proposed experimental design scheme (see Figure 2) is to determine the best values of the defined parameters in order to obtain the optimal control policies. First, we employ response surface design to generate sufficient experimental data points \((s, S, T)\), which are input for the simulation model. The output \(y\) of running the simulation model is the per unit time average cost. Then, we can fit the multiple regression models between the quantitative factors (policy parameters) and the response (cost performance of the policy). Finally, we use response optimizer embedded in Minitab to find optimal factor setting \((s^*, S^*, T^*)\). In the following subsections, the details of each block will be discussed.

**6.1 Simulation Model**

We build the simulation model using modules from the Blocks and Elements panels of ARENA which uses SIMAN simulation language (Kelton 2003). The modules are divided into three sections, as indicated by the outline boxes behind the modules.

The data points generated by the experimental design are inputs of the simulation model. The output of running the simulation model is the cost, which is the response variable of the following experimental design and analysis.
6.2 Response Surface Experimental Design

In this section, we employ the most commonly used response surface design, Central Composite Design (CCD) to generate sufficient experimental data to fit the second order effect, that is, curvature in the response surface. Compared with the three level factorial experimental design $3^k$ and fractional factorial experimental design $3^{k-p}$ designs, it uses a significantly fewer number of runs. To fit the second order effect, Central Composite design is most appropriate (Meyers et al. 1995). The factors of interest are independent variables $s$, $S$, and $T$. The response variable is the per unit time average cost. To ensure adequate coverage of the experimental region of interest, we chose the levels of each factor from the observation of the inventory levels of components and machine ages trajectory given by some preliminary runs made off-line. In addition, we achieved the rotatable CCD by setting $\alpha = (n_F)^{1/4}$, in which $\alpha$ is the distance from the center of the design space to an axial point and $n_F$ is the number of experimental runs in the factorial portion of the CCD. Rotatable CCD provides the desirable property of constant prediction variance at all points that are equidistant from the design center, thus improving the quality of the prediction. Table 1 shows the design parameters, data points, factors, and levels of each factor for the rotatable CCD.

<table>
<thead>
<tr>
<th>Basic Parameters</th>
<th>Factors: 3</th>
<th>Replicates: 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Base runs: 20</td>
<td>Total runs: 80</td>
</tr>
<tr>
<td></td>
<td>Base blocks: 1</td>
<td>Total blocks: 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Data Points</th>
<th>Cube points: 32</th>
<th>Center points in cube: 24</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Axial points: 24</td>
<td>Alpha: 1.68179</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Factors</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Little s$</td>
<td>0 10 15 30 45</td>
</tr>
<tr>
<td>$Big S$</td>
<td>33 50 75 100 117</td>
</tr>
<tr>
<td>Machine Age $T$</td>
<td>25 70 135 200 244</td>
</tr>
</tbody>
</table>

Table 1 Rotatable central composite design
The ANOVA table for the experimental data is shown in Table 2, and summarizes the linear terms, the squared terms, and the interactions. The small P-values for interaction (P-value ≈ 0.000) and the squared terms (P ≈ 0.000) suggest that there is significant curvature in the response surface, which makes the quadratic model reasonable. For the full quadratic model, the P-value for lack of fit is 0.132 suggesting that this model adequately fits the data. Residual plots shown in Figure 4 further indicate that the model is valid and adequate. The normal probability plot seems like a straight line indicating that the normal assumption is satisfied. The plot of residual versus the fitted values shows no obvious structure indicating the constant variance assumption is also satisfied.
The full quadratic model form is:

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_i x_i^2 + \sum_{i<j} \beta_{ij} x_i x_j + \varepsilon$$

(11)

By using least squares fit for multiple linear regressions, the least squares estimator of $\beta$ is:

$$\hat{\beta} = (X'X)^{-1}X'y$$

(12)

The matrix form of the fitted quadratic model can be expressed as:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_b' x + \hat{\beta}_B x$$

(13)

By solving $\frac{\partial y}{\partial x} = 0$, we can obtain the stationary point is $x_0 = -\frac{1}{2} B^{-1} b$. From Equation 12, the optimal cost $y$ is derived.

The regression coefficient estimates for all the terms in the quadratic model and the corresponding P-values are presented in Table 3. Because we used an orthogonal design, each effect is estimated independently. In the table, we can see small P-values for the interaction little $s$ by $T$ interaction ($p=0.001$) and the interaction big $S$ by $T$ interaction ($p=0.000$) suggesting these effects may be important. The contour plots and surface plots of cost versus factors are shown in Figure 5. These plots indicate that the lowest cost is obtained when $s$ and $S$ levels are low and $T$ levels are high. In addition, one can see the shape of the response surface and get a general idea of cost at various settings of $s$, $S$ and $T$. 

### Table 3 Regression surface regression coefficient estimation

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>SE Coefficient</th>
<th>T</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>48.508</td>
<td>0.3663</td>
<td>132.413</td>
<td>0.000</td>
</tr>
<tr>
<td>Little $s$</td>
<td>1.715</td>
<td>0.4088</td>
<td>4.195</td>
<td>0.000</td>
</tr>
<tr>
<td>Big $S$</td>
<td>11.846</td>
<td>0.4088</td>
<td>28.980</td>
<td>0.000</td>
</tr>
<tr>
<td>$T$</td>
<td>-4.111</td>
<td>0.4088</td>
<td>-10.057</td>
<td>0.000</td>
</tr>
<tr>
<td>Little $s$*Little $s$</td>
<td>1.401</td>
<td>0.6692</td>
<td>2.093</td>
<td>0.040</td>
</tr>
<tr>
<td>Big $S$*Big $S$</td>
<td>4.740</td>
<td>0.6692</td>
<td>7.083</td>
<td>0.000</td>
</tr>
<tr>
<td>$T$*$T$</td>
<td>4.043</td>
<td>0.6692</td>
<td>6.041</td>
<td>0.000</td>
</tr>
<tr>
<td>Little $s$*Big $S$</td>
<td>-1.280</td>
<td>0.8982</td>
<td>-1.425</td>
<td>0.159</td>
</tr>
<tr>
<td>Little $s$*$T$</td>
<td>-3.074</td>
<td>0.8982</td>
<td>-3.42</td>
<td>0.001</td>
</tr>
<tr>
<td>Big $S$*$T$</td>
<td>5.118</td>
<td>0.8982</td>
<td>5.69</td>
<td>0.000</td>
</tr>
</tbody>
</table>

$$S = 1.796$$  
$$R-Sq = 93.9\%$$  
$$R-Sq (adj)= 93.2\%$$

### 6.3 Experimental Data Analysis

The full quadratic model form is:

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_i x_i^2 + \sum_{i<j} \beta_{ij} x_i x_j + \varepsilon$$

(11)

By using least squares fit for multiple linear regressions, the least squares estimator of $\beta$ is:

$$\hat{\beta} = (X'X)^{-1}X'y$$

(12)

The matrix form of the fitted quadratic model can be expressed as:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_b' x + \hat{\beta}_B x$$

(13)

By solving $\frac{\partial y}{\partial x} = 0$, we can obtain the stationary point is $x_0 = -\frac{1}{2} B^{-1} b$. From Equation 12, the optimal cost $y$ is derived.

The regression coefficient estimates for all the terms in the quadratic model and the corresponding P-values are presented in Table 3. Because we used an orthogonal design, each effect is estimated independently. In the table, we can see small P-values for the interaction little $s$ by $T$ interaction ($p=0.001$) and the interaction big $S$ by $T$ interaction ($p=0.000$) suggesting these effects may be important. The contour plots and surface plots of cost versus factors are shown in Figure 5. These plots indicate that the lowest cost is obtained when $s$ and $S$ levels are low and $T$ levels are high. In addition, one can see the shape of the response surface and get a general idea of cost at various settings of $s$, $S$ and $T$. 

208
Figure 5 Contour plots and surface plots of cost versus factors $s$, $S$ and $T$

![Contour Plots for all pairs of factors](image1)

![Surface Plots for all pairs of Factors](image2)

Figure 6 Response optimization solution

<table>
<thead>
<tr>
<th>Optimal D</th>
<th>Little s</th>
<th>Big S</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99439</td>
<td>40.2269</td>
<td>117.0448</td>
<td>244.3165</td>
</tr>
<tr>
<td>0.09439</td>
<td>[10.0]</td>
<td>[50.0]</td>
<td>[223.5061]</td>
</tr>
</tbody>
</table>

The starting point $(s,S,T)$ is $(10,50,70)$. The response optimization solution $(s^*, S^*, T^*)=(10, 50, 223.5061)$ is shown in Figure 6. Therefore, the optimal control policy is that at each demand arrival epoch, if the inventory position of a component is less than 10, expedite the component, and the optimal expediting quantity is given by Equation(4); if the inventory position of a component is greater than 10 and less than 50, produce the component, and the optimal production quantity is given by Equation(3); if the inventory position of a component is greater than 50 and the machine age is less than 223, do nothing; and if the inventory position of the component is greater than 50 and the machine age is greater than 223, carry out preventive maintenance. The optimal per unit time average cost is 40.0561.

7 Conclusion

In this paper, we consider an unreliable multiple-product production/inventory system consisting of $N$ production machines and an assembly machine. We formulate the optimal control of the unreliable production/inventory system as a semi-Markov decision problem.
7.1 Contributions

1) By the separation principle, we simplify the control problem of unreliable multiple-product production/inventory system into the control problem of single product/single failure-prone machine.

2) We present a three-critical-limit policy that addresses the production/expediting/inventory and preventive maintenance controls in an integrated manner.

3) We propose a control approach and develop a response surface based experimental design framework to identify the optimal parameter values. The numerical results show the proposed control approach can be flexibly operated by changing the parameter settings. The parameter values can be optimized using simulation. Therefore, it provides an important guideline and insight for management to develop control policies.

4) Plambeck (Plambeck et al. 2003) proves that multi-dimensional ATO control problems can be separated into single-item inventory control problems, and the optimal production and expediting policy for each component is independent of all other components under the assumption that expedited components have zero leadtime and reliable normal production facilities, which is an unrealistic assumption for the real world. However, unpredictable machine failures have been identified as the main cause of uncertainty in manufacturing systems. Based on this observation, our model attempts to bridge this gap.

7.2 Future work

The results presented in this paper provide a first step toward a better understanding of the structure of the optimal policy for an unreliable multiple-product production/inventory system consisting of $N$ production machines and an assembly machine. There are several avenues for future research. In particular, it would be highly challenging to consider systems with multiple products without separation.

References


