Supply Chain Modeling with Bayesian Networks

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In our study, we focus on supply chains where an upstream agent (manufacturer) produces goods for downstream agents (retailers) in a make-to-stock (MTS) manner. In this setting, players first attempt to estimate their non-private information parameters. They then try to determine rational inventory policies based on acquired information without any regard for coordination. Our goal is to facilitate BN algorithms and methods for efficient inference of information not directly available to a supply chain stage/player. Therefore, the proposed methodology can become quite valuable in coping with a variety of SC operational issues. We used two different cases to analyze different degree detail as well as the impact of different levels of information "visibility".

Keyword: Supply Chain, Bayesian networks, Inventory Management.

1. Introduction

In recent decades, the increasing trend towards more higher levels of product variety, global marketplaces, shorter product life cycles, and premium customer service is exerting more and more pressure on companies and their supply chains to execute operations more effectively and efficiently (Sahin and Robinson, 2002, 2005). This is leading to a growing interest towards supply chain management (SCM) initiatives that can better serve well informed customers that are needs with diversified. Wal-Mart and Dell are examples of companies that have been vastly successful in leading and implementing various management initiatives, that allowed their supply chains to be operationally cost efficient while being highly responsive (Zhang, 2004; Chopra and Meindl, 2002).

Vast majority of the SCM literature available to date for all levels of planning (i.e., strategic, tactical, and operational) focuses on *centralized* decision making with the assumption of an integrated supply chain (SC) and/or centralized information. A classic example is the multi-echelon supply chain (SC) inventory management model proposed by Clark and Scarf (1960) and all their extensions (Axaster 2002, 2003, Axaster et al. 2002, Wang et al. 2004). Although they have attracted a lot of attention due to their analytical tractability, and in some cases mathematical richness, centralized SCM techniques are hard to implement for the fact that most supply chains are not integrated, with most stages owned by different entities. Therefore, it is much more attractive to develop and apply decentralized control strategies that let each SC installation decide its decisions based on “local” information and maybe information shared by players from adjacent stages (Axsater, 2005).

Decentralized supply chains constitute a challenging class of environment to service the customers for several reasons (Kiekintveld et al. 2005). Firstly, companies face substantial uncertainty from different sources; demand (volume and mix), process (yield, transportation reliabilities and machine downtimes), supply (delivery and transportation reliabilities) (Lee and Billington, 1993). Secondly, the supply chain is highly dynamic and decisions made by seemingly distant SC players can have detrimental effect on many stages of the supply chain. The so called “bullwhip effect” in SCs is a perfect example (Lee et. al, 1997). Thirdly, all the players are self interested; they behave rationally while making decisions. However, irrespective of the complexity of the decentralized decision making environment, supply chain managers have no choice but to confront this complexity and still seek to make effective decisions.

In our study, we will focus on supply chains where an upstream agent (manufacturer) produces goods for downstream agents (retailers) in a make-to-stock (MTS) manner. The performance of the supply chain in this setting broadly depends on three factors; demand realized by the retailers, effectiveness of the manufacturers production process (or/and inventory policy), and the retailers’ inventory replenishment policies. While supplier would prefer the retailer to hold as much inventory as possible, the retailer prefers quick response from supplier and to hold little inventories. Given competitive behaviors, the problem is non trivial even when players know the system state (e.g., cost parameters, demand rate and inventory positions). We will study the policies of players in a two-stage decentralized serial supply chain that consists of one supplier and one retailer. In this setting, players first attempt to estimate non-private information parameters. They then try to determine rational inventory policies based on acquired information without any regard for coordination.

Our goal is to as a later player (manufacturer) in the supply chain answer the question “How can he get a better estimate of the state of the downstream supply chain?” One answer to this question is a model that facilitates the interdependencies between the players and their policies in a timely manner with Bayesian Networks (BN). Overall objective of the present study is to promote the use of Bayesian Networks to facilitate effective supply chain management, particularly at the operation level. The intention is to demonstrate its potential as an intuitive modeling technique, yet rich enough to offer attractive features not always achievable by other means. Therefore, we aim in this study to mostly derive efficient solutions to some operational level supply chain problems in uncertain and complex environments with the expressive power of BNs.

Section 2 reviews literature of major relevance to the current study. Section 3 provides a more detailed description of the Bayesian Networks. Section 4 gives more detail explanation of the methodology, case studies and their results. Finally, we present conclusion and future study.

2. Literature Review

Information such as demand, lead time, capacity, cost parameters of the players is critical for companies while they are making their planning activities for inventory, production, distribution and coordination. Companies in many cases have to make decisions with incomplete and distorted information that adversely affect their planning activities. Therefore, to mitigate these distortion effects are main concern of the companies today. One of the early studies related to the information distortion is from Forester (1961). Then three decades later, Sterman (1989) uses the beer distribution game to explore the behaviors of decision makers and demand distortion. Although there are companies' initiatives to mitigate this distortion affect, it was not until second part of 1990s to give theoretical back round and underlying principles of demand distortion. Lee (1997a) analyzed the demand amplification along the supply chain and named this amplification effect as bullwhip effect. There are also several attempts to quantify the bullwhip effect in different scenarios (Metters 1997, Chen et al. 2000[a], Chen et. al 2000[b]).

Another body of literature related to the analysis of economic value of sharing information within the supply chain without any coordination. Hariharan and Zipkin (1995) explore the effects of customer advance warnings on system performance with a single, commodity like product. Gavirneni et al. (1999) study the relationships between capacity, inventory, and information, as well as how they are affected by the retailer's policy and end-item demand distribution in a typical serial supply chain with different information sharing scenarios. Lee et al. (2000) differently study the value of information sharing when demands are correlated over time in serial supply chain. Their analysis suggest that higher the demand correlation over time, higher the value of information sharing. Besides the information sharing, there are many studies attempt to facilitate coordination in supply chain. Researchers use different aspects such as contracts, game theory, hierarchical planning and recently multi-agent approach.

The fundamental issues in traditional inventory management are when to order and how much to order. Decisions involving these two issues become more complicated when demand in uncertain and various cost trade-offs are considered. Furthermore this complexity increases with consideration of several participants of a company's supply and demand network. One of the earliest studies of supply chain inventory management, so-called multi echelon inventory problem, is by Clark and Scarf (1960). By relaxing many assumptions and extending to the to more complex supply chain structures, many results emerge to the extension to the Clark and Scarf study (Federgruen and Zipkin 1984; Eppen and Schrage, 1981; Diks and de Kok, 1998 and 1999; Graves 1996; Axsater et al., 2002).

Our study differs from the current literature from different perspectives. Firstly, the real demand or forecasting information are assumed to known by the players of the supply chain in most of the inventory management problems. In our study the players will try to infer about demand information or the other private information of the players such as on hand information or cost parameters while interacting with each other. There is no direct information sharing attempt from the players which is more realistic and practical in current supply chains. Secondly, we aim to incorporate domain knowledge to the inference and decision making process more elegant way. Current literature do not easily facilitate incorporation of domain knowledge and temporal data for effective inventory decision making problems. For example, the domain knowledge of any manufacturer that supplies the goods for a retailer plays a crucial role in the success of the whole chain. Different experts may give different probability models according to his/her own experience in a certain area. Finally, despite the suggested benefits and examples of successful implementation of information sharing in supply chain, most of the currently available schemes proposed for evaluation of value of information are narrowly focused. To the best of our knowledge, there is no general framework/study that discusses the value of SC information available at different levels of granularity/detail. Therefore, this study targets development of BN algorithms and methods for efficient inference of information not directly available to a supply chain stage/player can become quite valuable in coping with SC operational issues.

3. Bayesian Networks

We have used Bayesian Networks to overcome the aforementioned deficiencies of traditional methods for SCM. Bayesian networks are a combination of both probability and graph theory. Probability theory provides the mechanism to incorporate the parts together as a consistent whole and graph theory provides an effective interface allowing experts to model sets of variables and their interdependencies. Therefore, the fundamental idea behind the Bayesian networks is modularity in which a complex and uncertain system can be built by combining its parts. From the reported literature, one can also argue that BNs are easier to elicit and interpret. Bayesian networks also offer many other advantages over traditional methods (that attempt direct modeling of the complete joint distribution) such as compact representation of knowledge, powerful interpretation of the world while interacting, ability to handle noisy and/or ambiguous data, provisions for incorporation of domain knowledge and data, handling of incomplete data sets, and learning and inference algorithms for parameter estimation and derivation of causal relationships.

Bayesian networks are graphical models consisting of variables and cause-effect relations between them. A Bayesian network consists of directed acyclic graph that represent dependencies among variables. A directed graph combination of nodes which represent variables and directed edges describe relations between variables. Directed graph contain no cycles or path that lead to from a node to itself. Each node (variable) has a finite of exclusive states and each node is assumed to be conditionally independent of its non-descendants given its parents.

Bayesian networks built in independence assumption that results in dramatic reduction of number of joint probabilities. If there are n variables, the complete distribution is specified by 2^n-1 joint probabilities. Each node has a conditional probability distribution $P(X_i|Parents(X_i))$ that quantifies the effect of the parents on the node (if there is a link from X to Y, X is said to be parent of Y). Absence of any link indicates conditional independence. The graphical model in Figure 1 is Bayesian network, since all edges are directed and no cycles within the model. In this graph, there is no edge between nodes D and E so they are conditional independent: ($P(D|C,E) = P(D|C)$ and $P(E|C,D) = P(E|C)$).

Joint distribution of all variables represented as;

$$P(A,B,C,D,E) = P(A|B,C,D,E)P(B|A,C,D,E)P(C|A,B,D,E)P(D|A,B,C,E)P(E|A,B,C,D)$$

by conditional property,

$$P(A,B,C,D,E) = P(A)P(B)P(C|A,B)P(D|C)P(E|C)$$

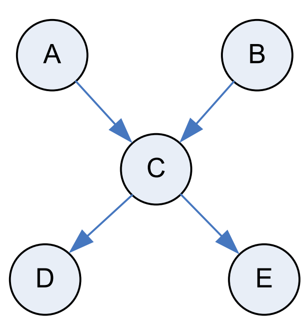

Figure 1. A simple Bayesian Network

3.1 Inference in Bayesian Networks

Basic task for any inference system is to compute posterior probability distribution for a query node given some observed nodes. We use capital letters *X, Y, Z* for variables, x, y, z to denote the specific values of these variables and boldface capital letters (**X, Y, Z**) as a set of variables. If *X* represents query node, **E** is set of observed nodes and **Y** is the set of nonobserved nodes. A typical query is $P(X|e) = \alpha P(X,e) = \alpha \sum_{y} P(X,e,y)$ where α is constant value independent from variable X. For illustration of inference, assume that all variables in Figure 1 are binary (true, false) variables and their conditional probabilities are given in Figure 2.

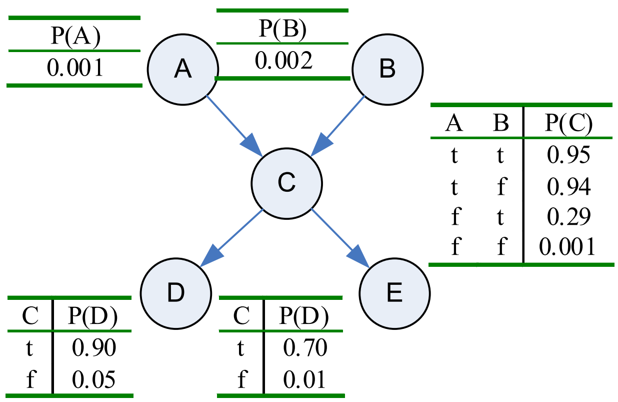

Figure 2. A typical Bayesian Network with conditional probability tables (CPts)

If we want to calculate probability of event A given the events D and E are true;

$$P(A|d,e) = \alpha P(A,d,e) = \alpha \sum_{b} \sum_{c} P(A,b,c,d,e)$$

$$P(A|d,e) = \alpha P(A,d,e) = \alpha(0.00059224, 0.0014919) = (0.284, 0.716)$$

4. Modeling One Supplier and Retailer Supply Chain with Bayesian Network

For illustrative purposes, we modeled a serial supply chain that consists of a retailer and manufacturer. The retailer and manufacturer employ an “updated *s*-*S* policy” and “order-up-to policy” for replenishment, respectively. The retailer meets the customer demand from inventory and any unsatisfied demand is backlogged. Manufacturer is supplied by an infinite capacitated supplier. Manufacturer satisfies the retailer orders from inventory and any unsatisfied retailer orders are backordered. One of the causes of bullwhip effect in SCs is due to batch ordering, as is the case in the current scenario. Here, the fact that we employ an updated *s*-*S* policy (at the retailer) hides true customer demand at the retailer from the manufacturer. In an *s*-*S* policy, when the inventory position falls below *s*, the system places an order to bring the inventory position to

S. In an "updated" s-S policy, the inventory position and the order up to points of the retailer are updated as follows;

$$s_t = avg(D)L + z\sigma\sqrt{L}$$

$$S_t = s_t k$$

where,

avg(*D*) is the moving average of demand faced by retailer over the past 10 periods

L denotes order lead time (lead time is one in all models)

z denotes a constant that is based on customer service level

σ denotes standard deviation of customer demand

k denotes a constant fixed during the model.

The purpose of employing an "updated" s-S policy is to provide manufacturer with a better belief about market demand and retailer state in different settings.

We model the above SC in different settings. In the first case, we assume that the manufacturer knows the true mean of the market demand but not the standard deviation. In the second case, we assume that the manufacturer knows the true standard deviation of market demand but not the mean. Our intention in both settings is to show the power of Bayesian Networks in learning the SC dynamics as well as its ability to infer true market demand and retailer policy parameters. Besides, we also study the effect of training sample size and different information sharing settings.

4.1. Case 1: Known Demand Mean and Unknown Demand Standard Deviation

In all settings, the manufacturer knows the inventory policy used by the retailer (updated s-S policy), his own parameters (order-up-to levels, lead time), and history of retailer orders. In all settings, we model true market demand as i.i.d. normal N(μ =20, σ^2 =64). The manufacturer's prior belief about the market demand is assumed N(20, 144). This prior information is very realistic because the long term average of orders converges to the true market demand in a serial supply chain. Simulation of the true market model with the updated s-S policy resulted in retailer orders with standard deviation of 15. The prior belief about variance of market demand (i.e., σ^2 = 144) is acceptable because of the existence of bullwhip effect. The comparisons of true "customer demand", true model simulation results of "retailer orders", and manufacturer prior belief about market demand are depicted in Figure 3. The existence of bullwhip affect is clear from the figure: the standard deviation of true market demand (Figure 3.a) is much higher than the standard deviation of retailer orders (Figure 3.b). The manufacturer prior belief about market demand (Figure 3.c) is close to the retailer' orders that are the only information the manufacturer can infer something about the downstream part of the supply chain. This validates the assumption of manufacturing belief about the variance of customer demand.

We modeled this serial supply chain as a BN under different settings. In particular, we study different degrees of detail as well as study the impact of different levels of information "visibility" on the performance of the BN. In all settings, firstly, we modeled the supply chain based on manufacturer's prior belief and determined the conditional probabilistic distributions between the variables. We then allow the BN to learn "on-line" based on the information visibility setting. Different inference algorithms are used with Expectation Maximization algorithm during the learning process.

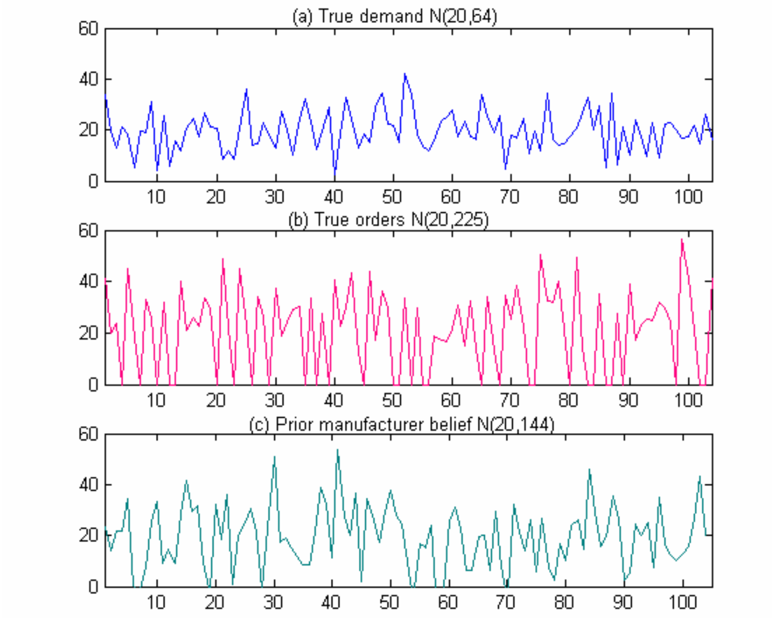

Figure 3. (a) True market demand, (b) Retailer orders, and (c)Manufacturer's prior belief about market demand.

Our first BN setting is composed of only two nodes; retailer's orders and customer demand as parent of orders (Figure 4a). In this setting, the manufacturer additionally knows the customer service level (z value in updated s equation) and order-up-to point parameter (k) of the retailer. The true market demand parameters, inventory availability at the retailer, true values of updated s and S, and backlog information is not known to the manufacturer in all settings.

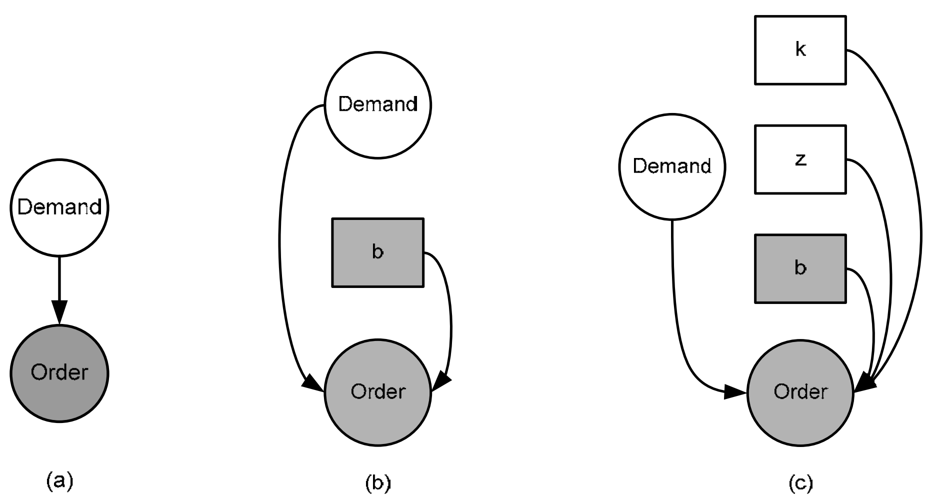

Figure 4. (a) Unknown demand; (b) Unknown demand with known backordering; (c) Unknown demand, customer service level and reorder inventory level with known backorder. (z: Customer service level parameter, k: order-up-to point parameter, b: backorder level. Clear nodes denote hidden variables and shaded observed variables. Square nodes denote discrete variables and circles continuous variables).

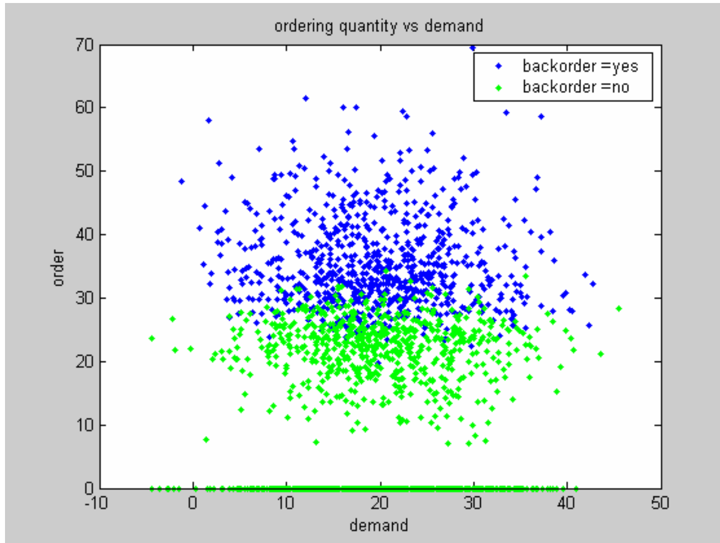

Figure 5. Customer demand vs. retailer orders with and without backorders.

In the second BN setting (Figure 4b), backorder node is added as a parent to the retailer orders node in addition to the demand node (Mixture of Factor Analysis). The backorder variable is a binary variable and represents the existence of backorders. The main advantage of this setting is to use the differentiating behavior of known manufacturing backorder information to infer for customer demand state. It is clear from Figure 5 that the average order quantity with backordering at the manufacturer is much higher than the case without backordering. In this BN, as in the first setting, the manufacturer knows the customer service level (z) and order-up-to point parameter (k) of the retailer.

In the third BN modeling setting (Figure 4c), manufacturer has some belief about the customer service level and order-up-to levels but he does not know the true values of the parameters. Therefore, we included the service level parameter (z: $z_1 < z_2$) with two different levels that dictate the actual reorder point and an order-up-to parameter (k: $k_1 < k_2$) with two different levels that dictate the actual order-up-to point. One of the levels for both parameters includes the true values (z_1 and k_1). Although we are accepting the parents of node "Order" (i.e., "k", "b", "z" and "demand") as independent variables, after observing the nodes "Order" and "b', all of these variables become dependent. We can conclude from the figure that the differentiating effect of order-up-to level parameter is more significant than the customer service level parameter. The model is Mixture of Factor Analysis and Figure 6 depicts the relation between order quantity and demand with different parameter settings. Order quantity with higher order-up-to point parameter (k_2) results in higher order quantities (42.6 to 39.7 on the average). Unlike parameter k, lower values of the customer service (z_2) results in higher order quantities (39.7 to 38.3 on the average). We can conclude that the differentiating effect of order-up-to point parameter on the order-demand relationship is more significant than the customer service level parameter.

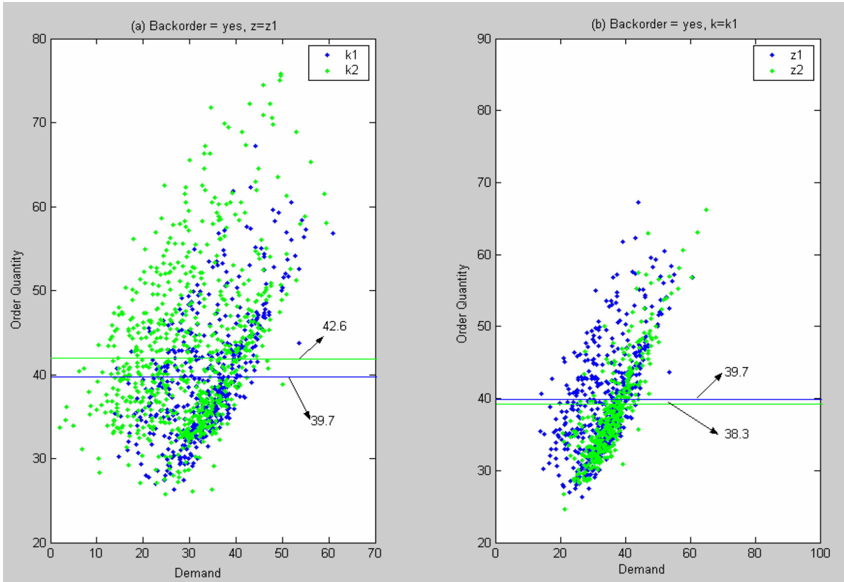

Figure 6. Customer demand vs. retailer orders with different customer service level and order-up-to parameters.

The learning results are depicted in Table 1. The model setting 1 performs poorly when compared to the true model. The differences between model setting 2 and model settings 3 with parameters (z_1, k_1) and (z_2, k_1) are very close. The other important observation from this table is related to the difference between the customer service level parameter (z) and order-up-to point parameter (k). If we compare standard deviation of model settings 3 with k_1 vs. k_2, and z_1 vs. z_2, we can see that the impact of parameter k is more significant than parameter z. This suggests that manufacturers should work harder to improve the quality of their prior belief for parameter k in comparison with parameter z. These results are similar with Figure 6a and 6b. We believe that this is mostly attributed to the applied ordering policy (updated s-S policy). Although the levels of z_1 and z_2 are selected so differently, the number of backlogs are quite similar for both levels.

Table 1. Results from BN modeling with known demand mean.

Model	Customer Demand Mean	Customer Demand Variance
True Model	*20*	*64*
Model Setting 1	*20.92*	*124.63*
Model Setting 2	*20.00*	*90.81*
Model Setting 3 (z_1,k_1)	*19.80*	*87.62*
Model Setting 3 (z_2,k_1)	*19.80*	*83.09*
Model Setting 3 (z_1,k_2)	*19.80*	*129.21*
Model Setting 3 (z_2,k_2)	*19.80*	*107.99*

4.2 **Case 2: Known Demand Standard Deviation and Unknown Demand Mean**

Unlikely the case of known demand mean, in the unknown mean demand setting we assume that the manufacturer knows the true demand standard deviation but has some belief about the true market demand mean. Here we modeled the SC as a Gaussian Mixture Model, different from the known demand mean case. The market demand mean type is included in the model as a discrete node. Therefore, the aim of the manufacturer is to estimate the any data point x belongs to demand type k and this is given by;

$$P(d=k \mid x,\theta)=\frac{\alpha_k P_k(x \mid \theta_k)}{\sum_j \alpha_j P_j(x \mid \theta_j)} \ .$$

The log-likelihood in this setting is given by;

$$\ell(\theta \mid D)=\sum_n \log(x^n)=\sum_n \log \sum_k \alpha_k N(x^n \mid \mu_k, \Sigma_k)$$

where μ_k denotes mean of the retailer orders for customer demand type k

Σ_k denotes covariance of the demand type k

x denotes the data point observable to the manufacturer

D denotes the whole data observable to the manufacturer

θ denotes the model parameters

Our objective is to evaluate the quality of Bayesian Network inference as well as the effects of sample size and different information sharing settings. The whole network is depicted in Figure 8. Shaded nodes denote variables observed by the manufacturer and square nodes denote discrete variables. As in the known mean demand case, the manufacturer can only observe backorders and orders. Since the retailer employs the (*S*,*s*) policy, the retailer does not orders in all periods/terms. Therefore, we introduce another binary variable that indicates whether or not an order is placed in a given term.

In our experiment, the manufacturer's prior belief about market demand consists of five different clusters; N(μ_1 =24, 1), N(μ_2 =22, 1), N(μ_3 =20, 1), N(μ_4 =18, 1) and N(μ_5 =16, 1) in which only one of them is the true market demand (Figure 7). Firstly, we trained the BN Model in Figure 8 with the belief of manufacturer with different sample sizes. We used 5, 25 and 100 as sample size in training for each combination of the demand type, customer service level factor, and order-up-to service level. Therefore, the total sample size for each training case is either 100 (2*2*5*5), 500, or 2000. We also measure quality of Bayesian Network inference with different information sharing settings. Firstly, we modeled the manufacturer as one that only observes orders, backorder level, and order type (shaded nodes in Figure 8). In the next setting, we allowed the manufacturer to access the order-up-to level factor (k). In the last setting, we also allowed the manufacturer to access the customer service level factor (z).

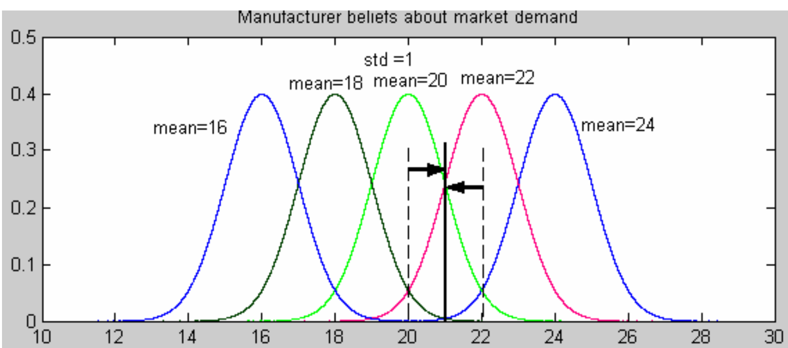

Figure 7. Manufacturer belief about market demand type with different means (standard deviation is always unity).

We used two different procedures to measure the accuracy of the model. In the first case, inference is based on a single data point. In the second case, inference is based on 5 data points at a time. These are respectively;

$$\arg\max_{1 \le k \le 5} \left[P(d = k \mid x, \theta) \right] \quad \text{and}$$

$$\arg\max_{1 \le k \le 5} \left[\prod_{j=1}^{5} P(d_j = k \mid x_j, \theta) \right].$$

Although the market demand is i.i.d., the orders given by the retailer to the manufacturer are not independent. However, within the static Bayesian Network, the assumption is that the data points are independent from case to case. It is for this reason that we also measure the accuracy of inference by using 5 consecutive data points at a time. In the future, these issues will be more effectively addressed by using Dynamic Bayesian Networks.

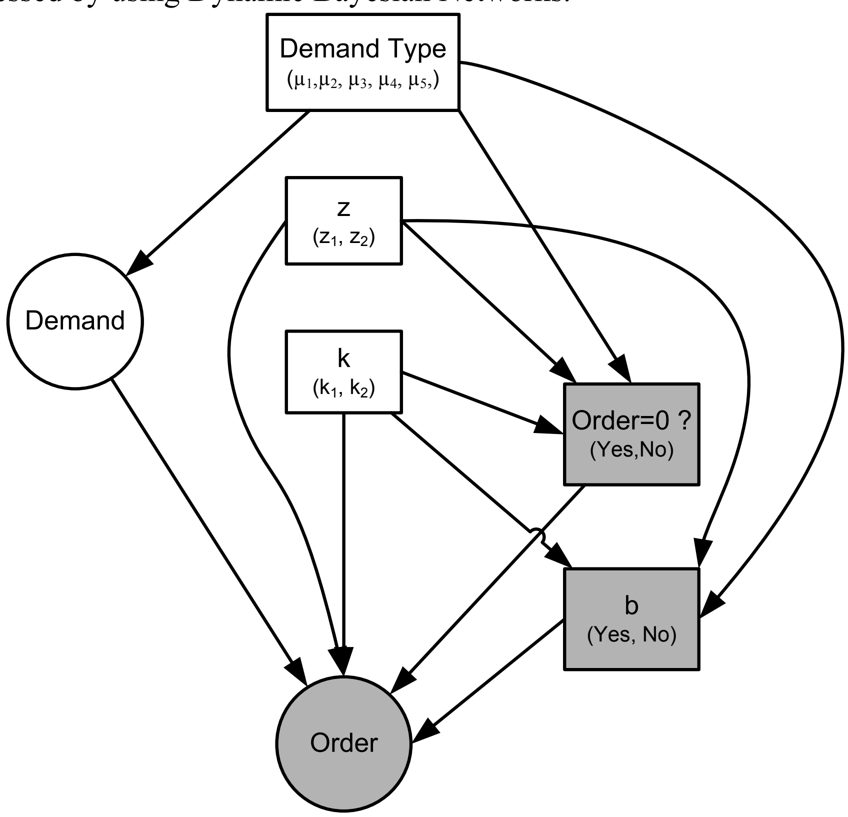

Figure 8. Bayesian network for the case of unknown mean customer demand (*z*: customer service level parameter, *k*: order-up-to point parameter, *b*: backorder).

The inference results from the Bayesian Networks are shown in Table 2. The left hand side mean values (μ_i) represent true values, and the values in each cell represent the probabilities that the whole sample data come from demand type i (upper side label, μ_i). Therefore, for each combination of factors (*k*, *z*, degree of observability, and number of cases for inference), we expect the highest probability in diagonal cells. Shaded cells represent the highest probability of the five different demand types in any setting. For example, in the first row, ([0.84 0.16 0 0 0]) denotes that the probability that retailer demand follows N(24,1) is 0.84 and the probability that demand follows N(22,1) is 0.16 after observing only the retailer orders and backorders when the true market demand is N(24,1). From the table, it is clear that the BN model performs rather well in many settings. The accuracy seems to increase with the sample size and the increment is higher from sample size 5 to 25 than 25 to 100. As expected, it is also clear from the table that inferences based on vectors of 5 data points are far more accurate than inferences based on 1 data point. Furthermore, the additional information accessed by the manufacturer does not improve the model. This is mostly related to the selected model parameters levels (k_1 vs. k_2 and z_1 vs. z_2), true model parameters, and used (s,S) policy. For example, since, the true known standard deviation is only 1 in all settings we would not expect very high number of backlogs at retailer without considering customer service level. We can conclude that the differentiating effect of customer service level factor on demand type is not important in our settings. This is also true for order-up-to level parameter; the levels of order-up-to level parameter (k_1 vs k_2) are selected so closely that it does not change the ordering patterns very much. In other words, at both levels, the number of consecutive days without ordering is very close.

5. Conclusion and Future Work

In this study, we successfully facilitate domain knowledge in SC Modeling, and the proposed methodology successfully implemented in inference of SC players' private information. Therefore, the proposed methodology provides quite valuable information that can be easily employed in coping with a variety of SC operational issues.

Although we observe some interesting and significant results, we believe that the accuracy of the model will increase with additional modeling effort. Firstly, we will take advantage of the sequential structure of data by modeling as a temporal model (DBNs). Secondly, the exact relationship between the variables will increase the accuracy of the model since we only incorporate some of the variables and their relations to the model. These additional modeling efforts should also help us in modeling more complex supply chain structures and independencies.

Table 2. Results from Bayesian Network modeling for the case of unknown mean demand type

Training Sample Size 5

Observability			Order, Backorder (b)									
Number of cases for inference			Inference for 1 case					Inference for 5 case				
			μ1	μ2	μ3	μ4	μ5	μ1	μ2	μ3	μ4	μ5
k1	z1	μ1	0.84	0.16	0.00	0.00	0.00	0.93	0.07	0.00	0.00	0.00
		μ2	0.54	0.4	0.05	0.00	0.00	0.00	1.00	0.00	0.00	0.00
		μ3	0.50	0.05	0.31	0.14	0.00	0.00	0.08	0.82	0.10	0.00
		μ4	0.50	0.00	0.00	0.40	0.10	0.00	0.00	0.00	0.87	0.13
		μ5	0.50	0.00	0.00	0.00	0.5	0.00	0.00	0.00	0.00	1.00
	z2	μ1	0.85	0.15	0.00	0.00	0.00	0.90	0.10	0.00	0.00	0.00
		μ2	0.55	0.4	0.04	0.00	0.00	0.05	0.95	0.00	0.00	0.00
		μ3	0.50	0.05	0.27	0.18	0.00	0.00	0.05	0.78	0.17	0.00
		μ4	0.50	0.00	0.00	0.36	0.14	0.00	0.00	0.00	0.93	0.07
		μ5	0.50	0.00	0.00	0.00	0.5	0.00	0.00	0.00	0.00	1.00
k2	z1	μ1	0.85	0.14	0.01	0.00	0.00	0.85	0.08	0.07	0.00	0.00
		μ2	0.74	0.24	0.01	0.00	0.00	0.50	0.4	0.07	0.00	0.00
		μ3	0.67	0.04	0.21	0.08	0.00	0.15	0.33	0.47	0.05	0.00
		μ4	0.55	0.10	0.01	0.30	0.05	0.00	0.07	0.33	0.60	0.00
		μ5	0.55	0.10	0.01	0.01	0.34	0.00	0.03	0.03	0.02	0.92
	z2	μ1	0.87	0.11	0.02	0.00	0.00	0.85	0.07	0.08	0.00	0.00
		μ2	0.68	0.30	0.02	0.00	0.00	0.33	0.58	0.08	0.00	0.00
		μ3	0.60	0.08	0.24	0.09	0.00	0.03	0.37	0.57	0.03	0.00
		μ4	0.53	0.08	0.01	0.32	0.06	0.00	0.02	0.37	0.60	0.02
		μ5	0.53	0.1	0	0	0.4	0	0.1	0	0	0.9

Observability			Order, Backorder (b), Order up to Level (k)									
Number of cases for inference			Inference for 1 case					Inference for 5 case				
			μ1	μ2	μ3	μ4	μ5	μ1	μ2	μ3	μ4	μ5
k1	z1	μ1	0.99	0.01	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
		μ2	0.61	0.34	0.05	0.00	0.00	0.08	0.92	0.00	0.00	0.00
		μ3	0.50	0.09	0.32	0.09	0.00	0.00	0.55	0.42	0.03	0.00
		μ4	0.50	0.00	0.01	0.38	0.12	0.00	0.00	0.05	0.77	0.18
		μ5	0.50	0.00	0.00	0.00	0.5	0.00	0.00	0.00	0.00	1.00
	z2	μ1	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
		μ2	0.61	0.35	0.04	0.00	0.00	0.12	0.88	0.00	0.00	0.00
		μ3	0.50	0.11	0.27	0.12	0.00	0.00	0.67	0.28	0.05	0.00
		μ4	0.50	0.00	0.00	0.34	0.2	0.00	0.00	0.07	0.73	0.20
		μ5	0.50	0.00	0.00	0.00	0.5	0.00	0.00	0.00	0.00	1.00
k2	z1	μ1	0.71	0.28	0.00	0.01	0.00	0.32	0.63	0.00	0.05	0.00
		μ2	0.69	0.30	0.00	0.01	0.00	0.25	0.7	0.00	0.05	0.00
		μ3	0.67	0.02	0.15	0.10	0.07	0.17	0.20	0.33	0.30	0.00
		μ4	0.55	0.10	0.00	0.33	0.02	0.00	0.07	0.28	0.65	0.00
		μ5	0.55	0.10	0.00	0.21	0.14	0.00	0.03	0.00	0.72	0.25
	z2	μ1	0.60	0.38	0.00	0.02	0.00	0.08	0.83	0.00	0.08	0.00
		μ2	0.63	0.36	0.00	0.02	0.00	0.13	0.78	0.00	0.08	0.00
		μ3	0.59	0.03	0.23	0.09	0.06	0.03	0.20	0.48	0.28	0.00
		μ4	0.53	0.08	0.00	0.36	0.03	0.00	0.02	0.27	0.72	0.00
		μ5	0.5	0.1	0	0.3	0.2	0	0.1	0	0.8	0.1

Observability			Order, Backorder (b), Order up to Level (k), Customer Service Level (z)									
Number of cases for inference			Inference for 1 case					Inference for 5 case				
			μ1	μ2	μ3	μ4	μ5	μ1	μ2	μ3	μ4	μ5
k1	z1	μ1	0.97	0.03	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
		μ2	0.51	0.44	0.05	0.00	0.00	0.02	0.98	0.00	0.00	0.00
		μ3	0.50	0.06	0.31	0.13	0.00	0.00	0.4	0.42	0.17	0.00
		μ4	0.50	0.00	0.00	0.42	0.08	0.00	0.00	0.00	0.90	0.10
		μ5	0.50	0.00	0.00	0.00	0.5	0.00	0.00	0.00	0.00	1.00
	z2	μ1	0.86	0.14	0.00	0.00	0.00	0.50	0.5	0.00	0.00	0.00
		μ2	0.63	0.35	0.02	0.00	0.00	0.08	0.92	0.00	0.00	0.00
		μ3	0.50	0.20	0.30	0.00	0.00	0.00	0.78	0.22	0.00	0.00
		μ4	0.50	0.02	0.18	0.11	0.19	0.00	0.05	0.33	0.03	0.58
		μ5	0.50	0.00	0.00	0.00	0.5	0.00	0.00	0.00	0.00	1.00
k2	z1	μ1	0.29	0.71	0.00	0.00	0.00	0.70	0.30	0.00	0.00	0.00
		μ2	0.27	0.56	0.09	0.08	0.00	0.47	0	0.27	0.25	0.00
		μ3	0.14	0.57	0.18	0.01	0.09	0.27	0.00	0.68	0.05	0.00
		μ4	0.03	0.65	0.17	0.00	0.15	0.03	0.07	0.73	0.17	0.00
		μ5	0.18	0.55	0.05	0.05	0.17	0.12	0.10	0.00	0.55	0.23
	z2	μ1	0.60	0.38	0.00	0.02	0.00	0.32	0.65	0.02	0.02	0.00
		μ2	0.54	0.4	0.00	0.02	0.00	0.00	0.92	0.07	0.02	0.00
		μ3	0.54	0.08	0.23	0.15	0.00	0.00	0.38	0.50	0.12	0.00
		μ4	0.53	0.08	0.01	0.38	0.00	0.00	0.28	0.08	0.63	0.00
		μ5	0.5	0.1	0	0.3	0.1	0	0.1	0.1	0.9	0

Table 2. (cont...) Results from Bayesian Network modeling for the case of unknown mean demand type.

			Training Sample Size 25																													
Observability			Order, Backorder (b)										Order, Backorder (b), Order up to Level (k)										Order, Backorder (b), Order up to Level (k), Customer Service Level (z)									
Number of cases for inference			Inference for 1 case					Inference for 5 case					Inference for 1 case					Inference for 5 case					Inference for 1 case					Inference for 5 case				
			μ_1	μ_2	μ_3	μ_4	μ_5	μ_1	μ_2	μ_3	μ_4	μ_5	μ_1	μ_2	μ_3	μ_4	μ_5	μ_1	μ_2	μ_3	μ_4	μ_5	μ_1	μ_2	μ_3	μ_4	μ_5	μ_1	μ_2	μ_3	μ_4	μ_5
k1	z1	μ_1	0.48	0.52	0.00	0.00	0.00	0.97	0.03	0.00	0.00	0.00	0.98	0.02	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.97	0.03	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
		μ_2	0.01	0.96	0.03	0.00	0.00	0.00	0.97	0.03	0.00	0.00	0.51	0.5	0.02	0.00	0.00	0.00	0.98	0.02	0.00	0.00	0.50	0.5	0.03	0.00	0.00	0.00	0.97	0.03	0.00	0.00
		μ_3	0.00	0.51	0.5	0.02	0.02	0.00	0.02	0.98	0.00	0.00	0.50	0.01	0.48	0.01	0.00	0.00	0.02	0.98	0.00	0.00	0.50	0.01	0.5	0.02	0.00	0.00	0.02	0.98	0.00	0.00
		μ_4	0.00	0.50	0.00	0.39	0.11	0.00	0.00	0.00	0.90	0.10	0.50	0.00	0.05	0.43	0.02	0.00	0.00	0.12	0.85	0.03	0.50	0.00	0.06	0.40	0.04	0.00	0.00	0.15	0.78	0.07
		μ_5	0.00	0.50	0.01	0.00	0.49	0.00	0.00	0.02	0.00	0.98	0.50	0.00	0.01	0.00	0.49	0.00	0.00	0.00	0.00	1.00	0.50	0.00	0.01	0.00	0.5	0.00	0.00	0.00	0.00	1.00
	z2	μ_1	0.48	0.51	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.99	0.01	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.99	0.01	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
		μ_2	0.03	0.95	0.02	0.00	0.00	0.03	0.95	0.02	0.00	0.00	0.54	0.4	0.02	0.00	0.00	0.03	0.97	0.00	0.00	0.00	0.56	0.4	0.02	0.00	0.00	0.07	0.93	0.00	0.00	0.00
		μ_3	0.00	0.50	0.42	0.03	0.04	0.00	0.00	1.00	0.00	0.00	0.50	0.01	0.46	0.03	0.00	0.00	0.00	1.00	0.00	0.00	0.50	0.01	0.5	0.03	0.00	0.00	0.00	1.00	0.00	0.00
		μ_4	0.00	0.50	0.01	0.41	0.08	0.00	0.00	0.02	0.93	0.05	0.50	0.00	0.04	0.43	0.02	0.00	0.00	0.08	0.92	0.00	0.50	0.00	0.05	0.4	0.03	0.00	0.00	0.08	0.92	0.00
		μ_5	0.00	0.50	0.02	0.01	0.48	0.00	0.00	0.00	0.02	0.98	0.50	0.00	0.01	0.01	0.48	0.00	0.00	0.00	0.03	0.97	0.50	0.00	0.00	0.01	0.5	0.00	0.00	0.00	0.03	0.97
k2	z1	μ_1	0.41	0.57	0.02	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.28	0.57	0.03	0.12	0.00	0.82	0.08	0.03	0.07	0.00	0.28	0.57	0.00	0.15	0.00	0.82	0.00	0.02	0.17	0.00
		μ_2	0.14	0.85	0.01	0.00	0.00	0.17	0.78	0.05	0.00	0.00	0.13	0.73	0.12	0.01	0.00	0.15	0.75	0.10	0.00	0.00	0.15	0.71	0.01	0.13	0.00	0.32	0.55	0.07	0.07	0.00
		μ_3	0.01	0.66	0.26	0.03	0.04	0.00	0.15	0.83	0.00	0.02	0.01	0.68	0.18	0.03	0.10	0.00	0.15	0.80	0.00	0.05	0.06	0.59	0.22	0.03	0.10	0.10	0.05	0.80	0.00	0.05
		μ_4	0.05	0.55	0.05	0.30	0.05	0.07	0.00	0.02	0.88	0.03	0.00	0.55	0.05	0.25	0.15	0.00	0.00	0.10	0.80	0.10	0.00	0.55	0.19	0.11	0.15	0.00	0.00	0.77	0.13	0.10
		μ_5	0.00	0.63	0.02	0.02	0.33	0.00	0.03	0.00	0.03	0.93	0.00	0.55	0.06	0.03	0.37	0.00	0.00	0.03	0.03	0.93	0.00	0.55	0.00	0.09	0.36	0.00	0.00	0.02	0.05	0.93
	z2	μ_1	0.45	0.53	0.02	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.34	0.53	0.02	0.11	0.00	0.90	0.02	0.02	0.07	0.00	0.34	0.64	0.02	0.00	0.00	0.82	0.10	0.08	0.00	0.00
		μ_2	0.10	0.88	0.02	0.00	0.00	0.10	0.82	0.08	0.00	0.00	0.09	0.77	0.12	0.02	0.00	0.08	0.75	0.17	0.00	0.00	0.09	0.79	0.11	0.00	0.00	0.08	0.82	0.10	0.00	0.00
		μ_3	0.00	0.61	0.36	0.00	0.02	0.00	0.05	0.95	0.00	0.00	0.00	0.61	0.25	0.00	0.14	0.00	0.03	0.93	0.00	0.03	0.00	0.61	0.25	0.00	0.14	0.00	0.13	0.83	0.00	0.03
		μ_4	0.06	0.53	0.02	0.33	0.06	0.02	0.00	0.07	0.88	0.03	0.00	0.53	0.03	0.30	0.14	0.00	0.00	0.12	0.80	0.08	0.00	0.61	0.02	0.23	0.14	0.00	0.05	0.20	0.67	0.08
		μ_5	0	0.6	0	0	0.4	0	0.1	0	0.1	0.9	0	0.5	0	0	0.4	0	0	0.1	0.1	0.9	0	0.5	0.1	0	0.4	0	0.1	0.1	0	0.9

			Training Sample Size 100																													
Observability			Order, Backorder (b)										Order, Backorder (b), Order up to Level (k)										Order, Backorder (b), Order up to Level (k), Customer Service Level (z)									
Number of cases for inference			Inference for 1 case					Inference for 5 case					Inference for 1 case					Inference for 5 case					Inference for 1 case					Inference for 5 case				
			μ_1	μ_2	μ_3	μ_4	μ_5	μ_1	μ_2	μ_3	μ_4	μ_5	μ_1	μ_2	μ_3	μ_4	μ_5	μ_1	μ_2	μ_3	μ_4	μ_5	μ_1	μ_2	μ_3	μ_4	μ_5	μ_1	μ_2	μ_3	μ_4	μ_5
k1	z1	μ_1	0.46	0.03	0.50	0.00	0.00	0.98	0.02	0.00	0.00	0.00	0.47	0.53	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.47	0.53	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
		μ_2	0.00	0.47	0.53	0.00	0.00	0.00	0.97	0.03	0.00	0.00	0.00	0.97	0.02	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.01	0.97	0.03	0.00	0.00	0.00	1.00	0.00	0.00	0.00
		μ_3	0.00	0.01	0.97	0.02	0.00	0.00	0.02	0.98	0.00	0.00	0.00	0.51	0.47	0.02	0.00	0.00	0.02	0.98	0.00	0.00	0.00	0.51	0.47	0.02	0.00	0.00	0.02	0.98	0.00	0.00
		μ_4	0.00	0.00	0.50	0.46	0.04	0.00	0.00	0.00	0.93	0.07	0.00	0.50	0.00	0.46	0.04	0.00	0.00	0.00	0.95	0.05	0.00	0.50	0.00	0.46	0.04	0.00	0.00	0.00	0.95	0.05
		μ_5	0.00	0.00	0.50	0.00	0.50	0.00	0.00	0.00	0.00	1.00	0.00	0.50	0.00	0.01	0.49	0.00	0.00	0.00	0.00	1.00	0.00	0.50	0.00	0.01	0.49	0.00	0.00	0.00	0.00	1.00
	z2	μ_1	0.48	0.02	0.50	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.48	0.52	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.48	0.02	0.00	0.00	0.50	1.00	0.00	0.00	0.00	0.00
		μ_2	0.01	0.46	0.52	0.00	0.00	0.03	0.95	0.02	0.00	0.00	0.02	0.96	0.02	0.00	0.00	0.03	0.97	0.00	0.00	0.00	0.03	0.46	0.02	0.00	0.50	0.03	0.97	0.00	0.00	0.00
		μ_3	0.00	0.00	0.96	0.03	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.51	0.46	0.03	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.01	0.46	0.03	0.50	0.00	0.00	1.00	0.00	0.00
		μ_4	0.00	0.00	0.50	0.46	0.04	0.00	0.00	0.00	0.93	0.07	0.00	0.50	0.01	0.46	0.03	0.00	0.00	0.00	0.97	0.03	0.00	0.00	0.01	0.46	0.53	0.00	0.00	0.00	0.98	0.02
		μ_5	0.00	0.00	0.50	0.01	0.49	0.00	0.00	0.00	0.00	1.00	0.00	0.50	0.00	0.01	0.49	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.02	0.98	0.00	0.00	0.00	0.02	0.98
k2	z1	μ_1	0.40	0.00	0.59	0.00	0.01	1.00	0.00	0.00	0.00	0.00	0.28	0.00	0.71	0.00	0.01	0.85	0.00	0.15	0.00	0.00	0.83	0.00	0.15	0.00	0.01	0.70	0.00	0.30	0.00	0.00
		μ_2	0.13	0.29	0.56	0.00	0.01	0.22	0.78	0.00	0.00	0.00	0.13	0.18	0.68	0.00	0.02	0.20	0.73	0.07	0.00	0.00	0.69	0.18	0.01	0.12	0.01	0.25	0.70	0.00	0.05	0.00
		μ_3	0.07	0.06	0.84	0.00	0.03	0.17	0.02	0.82	0.00	0.00	0.07	0.06	0.74	0.00	0.13	0.25	0.02	0.65	0.03	0.05	0.63	0.05	0.18	0.00	0.13	0.22	0.03	0.53	0.17	0.05
		μ_4	0.04	0.00	0.61	0.34	0.00	0.05	0.00	0.07	0.88	0.00	0.00	0.00	0.65	0.20	0.15	0.02	0.00	0.22	0.67	0.10	0.55	0.01	0.17	0.12	0.15	0.00	0.00	0.33	0.38	0.28
		μ_5	0.00	0.08	0.56	0.03	0.32	0.00	0.03	0.00	0.02	0.95	0.00	0.00	0.62	0.01	0.37	0.00	0.00	0.03	0.02	0.95	0.55	0.00	0.00	0.11	0.34	0.00	0.00	0.00	0.20	0.80
	z2	μ_1	0.45	0.00	0.54	0.00	0.02	1.00	0.00	0.00	0.00	0.00	0.34	0.00	0.64	0.00	0.02	0.90	0.00	0.10	0.00	0.00	0.34	0.00	0.11	0.00	0.55	0.92	0.00	0.08	0.00	0.00
		μ_2	0.10	0.35	0.54	0.00	0.02	0.15	0.83	0.00	0.02	0.00	0.09	0.23	0.64	0.00	0.03	0.13	0.78	0.05	0.02	0.02	0.09	0.23	0.12	0.00	0.55	0.12	0.77	0.10	0.00	0.02
		μ_3	0.03	0.04	0.92	0.00	0.00	0.03	0.00	0.97	0.00	0.00	0.03	0.04	0.79	0.00	0.14	0.08	0.00	0.88	0.00	0.03	0.03	0.03	0.26	0.00	0.68	0.08	0.03	0.85	0.00	0.03
		μ_4	0.05	0.00	0.56	0.39	0.00	0.02	0.00	0.02	0.97	0.00	0.00	0.00	0.61	0.25	0.14	0.00	0.00	0.12	0.78	0.10	0.00	0.00	0.08	0.24	0.68	0.00	0.00	0.22	0.65	0.13
		μ_5	0	0.1	0.5	0	0.4	0	0.1	0	0	0.9	0	0	0.6	0	0.4	0	0	0.1	0	0.9	0	0	0.1	0	0.9	0	0	0.1	0	1

References

Axsaster, S. (2000) Exact analysis of continuous review (R,Q) policies in two-echelon inventory systems with compound Poisson demand", Operation Research 48, 686-696.

Axsaster, S. (2002) "Heuristic Methods for Centralized Control of One-Warehouse, N-Retailer Inventory Systems", Manufacturing & Service Operations Management, 4, 75-97.

Axsaster, S. (2005) "A simple decision rule for decentralized two-echelon inventory control", International Journal of Production Economics, 93-94, 53-59

Chen F, Ryan J., and Simchi-Levi, D., (2000b) "The impact of exponential smoothing forecasts on the bullwhip effect", Naval Research Logistics, Vol. 47, 269-286.

Chen, F., Drezner, Z., Rayan, J., Simchi-Levi, D., (2000a)"Quantifying the bullwhip effect in a simple supply chain: the impact of forecasting, lead times, and information", Management Science, Vol. 46, No. 3, 436-443.

Chopra, S. and Meindl, P. (2002), Supply Chain Management: Strategy, Planning, and Operations, Pearson Education Asia.

Clark, A. J. and H. E. Scarf, (1960), “Optimal Policies for a Multi-echelon Inventory System”, Management Science, 5, 475-490

Diks, E. B., A.G. de Kok (1998) “Optimal control of a divergent multi-echelon inventory system”, European Journal of Operation Research, 111, 75-97.

Diks, E. B., A.G. de Kok (1999), “Computational results for the control of a divergent N-echelon inventory system”, International Journal of Production Economics, 59, 327-336.

Federgruen, A., and P. H. Zipkin (1984) “Computational issues in an infinite-horizon multi-echelon inventory model”, Operation Research 32, 818-836.

Forester J., (1961), Industrial Dynamics, MIT Press, Cambridge, MA, and Wiley, New York.

Gavirneni, S., R. Kapuscin, and S. Tayur, (1999), “Value of information in capacitated supply chains”, Management Science, 46(1):16-24.

Graves, S.C (1996) “A multiechelon inventory model with fixed replenishment Intervals”, Management Science, 42, 1-18.

Hariharan, R., & Zipkin, P. (1995). “Customer-order information, lead-times, and inventories”, Management Science, 41(10), 1599-1607.

Kiekintveld C., Wellman M. P. , Singh S. P. , Estelle J., Vorobeychik Y., Soni V., Rudar M. R., (2004), “Distributed Feedback Control for Decision Making on Supply Chains” ICAPS 2004, 384-392

Lee H. L and Billington C. (1993) “Material management in decentralized supply chains”, Operation Research, 41, 835-847.

Lee, H,. P. Padmanabhan, S. Whang,. (1997a) “Information distortion in a supply chain: The bullwhip effect”, Management Science, 43, 546-558.

Lee, H,. P. Padmanabhan, S. Whang,. (1997a) “The bullwhip effect in supply chains”, Sloan Management Review, 38, 93-102

Lee, H., K. So, and C. Tang. (2000). “The value of information sharing in a two-level supply chain”, Management Science, 46(5):626-643.

Metters R., “Quantifying the bullwhip effect in supply chains”, Journal of Operations Management, Vol. 15, (1997), 89-100.

Sahin F., and Robinson Jr. E. P., (2002) “Flow coordination and information sharing in supply chains: review, implications, and directions for future research”, Decision Sciences, 33, 2002.

Sahin F., and Robinson Jr. E. P., (2005) “Information sharing and coordination in make to order supply chains”, Journal of Operations Management, 23, 579-598.

Sterman, J. D. (1989), Modeling managerial behavior: misperceptions of feedback in a dynamic deicison making experiment”, Management Science, 35 (3), 321-339

Xiaolong, Zhang (2004), “The impact of forecasting methods on the bullwhip effect”, International Journal of Production Economics, 88,15–27