Penalized Surrogate Subgradient Method Based on Lagrangian Relaxation

Yan Guo, Department of Industrial and Manufacturing Engineering, Wayne State University

The Lagrangian Relaxation Method has been successfully applied in unit commitment scheduling of power systems. However, the dual solution will be far from optimal and the solution will oscillate seriously if identical units exist in the system. As a result, the quality of the feasible solution obtained may be very unsatisfactory. This issue has been long recognized as an inherent disadvantage of Lagrangian Relaxation (LR) based methods. In this paper, the homogeneous solution issue is identified. A Penalized Surrogate Subgradient method (PSS) is developed. This new method combines the concepts of augmented LR and surrogate subgradient to produce a good research direction at a high level. The low level subproblems including those corresponding to the identical units are solved successively so that the commitments of the identical units may not be homogeneous in the dual solution. Compared with the Standard Lagrangian Relaxation method (SLR), the new method can obtain a better dual solution and avoid solution oscillations. Numerical testing results support the effects of this new method.

Keyword: Lagrangian Relaxation, Power System Scheduling, Unit Commitment, Surrogate Subgradient Algorithm.

1. Introduction

It is very profitable to optimize the unit commitment scheduling problems in power systems. But because of the complex of problems, it is a hot spot in the application of optimization. Many methods have been developed. One of the most successful applications of SLR method is in unit commitments (Baldick 1995; Cohen and Sherkat 1987; Ferreira et al. 1989; Guan et al. 1992; Shaw 1995; Wang et al. 1995), which are important problems in determining the commitment and generation levels of generating units over a period of time to minimize the total operation cost. However, a serious and inherent issue of SLR is the solution oscillation caused by the homogeneous subproblems solutions. When the units in a system are identical or similar, the dual solution is far from the optimal solution of primal problem. Until now, this kind of oscillation - isoperimetric problem - has not been avoided.

Isoperimetric problems exist because the SLR method is a price coordination method; therefore solutions of the subproblems associated with identical units are the same. The lagrangian multiplier reflects the marginal price. Under the LR frame, subproblems are the same because of the same parameters of identical units in a system. No matter how the multiplier is adjusted, the dual solutions of the subproblems of the identical units are the same. If there are some discrete decision variables (e.g. the variables of unit status representing unit turning on or off), the slight change of multiplier may make these variables change together. For example, there are two identical units in a generating system. The optimal solution is only one of them working at some time. However, the actual optimal solution obtained by SLR is that either both of them work, or both of them do not work. This kind of isoperimetric problem leads to an outcome that the dual solution will be far from the optimal solution of the primal problem. In other words, the dual solution is not precise enough to get the optimal solution so as to achieve the quality of a feasible solution.

A surrogate subgradient method is developed to solve the thermal generation scheduling problem (Cohen and Sherkat 1987). Although the dual solution can be found quickly, the oscillation will happen because of the identical units when solving the low level subproblems. And a parameter perturbation method was employed (Lai F 1999) to differentiate the subproblems by using some approaches to change parameters. However, in fact, the primal problem was changed as well. With the augmented LR method, the decomposition of subproblems was destroyed because of the correlation introduced by penalty factor. Though this problem can be solved theoretically, the combination explosion will arise. For instance, in one scheduling project with a 4-hour scheduling period, 10 units are in the system; each unit's minimum up time and down time are both 2 hours. All units' initial statuses are off, and all units have been off for enough hours to meet the minimum down time. Then there are 11 combinations of each unit according to the state transition diagram (Guan et al. 1992). For ten units, the possible combination is 11^{10}. This is a tiny example, but the combination is a huge number. Therefore, when the penalty factor is introduced to make the problem indecomposable, it is failed yet avoided oscillation by linearizing the penalty term (Wang et al. 1995).

2. Problem Formulation

According to above disadvantages and limitation, in this paper, a new method based on LR is given. This method uses a penalty factor multiplied to system demand and system spinning reserve constraints. This penalty factor destroy the decomposability of the problem, which makes it difficult to solve subproblems. However, if the surrogate subgradient method in (Zhao et al. 1999) is combined, the decomposability of subproblems will be obtained again by using a surrogate subgradient to adjust the direction of the multiplier. One of the important intentions of solving subproblems under SLR frame is to find the direction to adjust the multiplier. This method is still effective when the penalty factor is introduced by treating the variables related to other units as constants. Therefore, if the subproblems are solved successively, no matter if the multiplier changes or not, the term of other variables in the penalty term has changed so that the cost function is changed differently, which avoids the oscillation. And another advantage is the violation degree to constraints will be decreased, which influences the quality of the feasible solution finally.

Considering a power system which has i units with ramping constrains, the scheduling objective is to find the status and the generation level of each unit in a scheduling period T to minimize the cost, and at the same time, the constraints should be satisfied ((Cohen and Sherkat 1987)). The model of the scheduling problem is as follow:

Objective Function:

$$\min_{\substack{u_i(t-1)\\ p_i(t)}} J ,$$

$$J \equiv \sum_{t=1}^{T} \sum_{i=1}^{I} \left[C_i\left(p_i(t)\right) + S_i\left(x_i(t-1), u_i(t-1)\right) \right] \quad (1)$$

Systems Constraints:

a. Constraint of system demand at time t:

$$\sum_{i=1}^{I} p_i(t) = P_d(t) \quad t = 1, 2, \cdots, T \quad (2)$$

b. Constraint of system spinning reserve requirement at time t:

$$\sum_{i=1}^{I} r_i(x_i(t), p_i(t)) \geq P_r(t) \quad t = 1, 2, \cdots, T \quad (3)$$

Where $p_i(t)$ is the generation levels, $x_i(t)$ the status of unit i at time t, $r_i\left(x_i(t), p_i(t)\right)$ the system spinning reserve contribution, $S_i\left(x_i(t), u_i(t)\right)$, $C_i\left(p_i(t)\right)$ the setup cost and fuel cost per hour of unit i. $P_d(t)$, $P_r(t)$ the system demand and spinning reserve at time t.

Single Unit Constraints(Wang et al. 1995):

c. State transition:

$$x_i(t+1) = x_i(t) + u_i(t) \quad if \quad x_i(t)u_i(t) > 0$$
$$x_i(t+1) = u_i(t) \quad if \quad x_i(t)u_i(t) < 0$$

d. Unit commitment

$$\underline{P_i} \leq p_i(t) \leq \overline{P_i} \quad if \quad x_i(t) > 0$$
$$p_i(t) = 0 \quad if \quad x_i(t) < 0$$

e. Minimum up/down time of unit *j*

$$u_i(t) = 1 \quad if \quad 1 \leq x_i(t) < \overline{\tau_i}$$
$$u_i(t) = -1 \quad if \quad -\underline{\tau_i} < x_i(t) \leq -1$$

f. Ramping constraint:

$$\left| p_i(t+1) - p_i(t) \right| \leq \Delta_i$$
$$if \quad x_i(t) > 0, x_i(t+1) > 0$$

g. Must be turn on or off:

$$x_i(t) > 0 \quad if \quad t \in mustup\{i\}$$
$$x_i(t) < 0 \quad if \quad t \in mustdown\{i\}$$

3. Solution Methodology

The solution is still based on Lagrangian relaxation. $\lambda(t)$ and $\mu(t)$ ($\mu(t) \geq 0$) are multipliers of systems constraints (2) and (3). $w > 0$ is the penalty factor.

$$
\begin{aligned}
&L\left(\lambda, \mu, p_i(t), u_i(t-1), w\right) \\
&=\sum_{i=1}^{I}\sum_{t=1}^{T}\left[C_i\left(p_i(t)\right)+S_i\left(x_i(t-1), u_i(t-1)\right)\right] \\
&+\sum_{t=1}^{T}\lambda(t)\left[P_d(t)-\sum_{i=1}^{I}p_i(t)\right] \\
&+\sum_{t=1}^{T}\mu(t)\left[P_r(t)-\sum_{i=1}^{I}r_i\left(x_i(t), p_i(t)\right)\right] \\
&+w\left(\sum_{t=1}^{T}\left|\left[P_d(t)-\sum_{i=1}^{I}p_i(t)\right]\right|\right) \\
&+w\left(\sum_{t=1}^{T}\left|\min\left\{0\ ,\ \sum_{i=1}^{I}r_i\left(x_i(t), p_i(t)\right)-P_r(t)\right\}\right|\right)
\end{aligned} \tag{4}
$$

The dual problem is followed:

$$
\begin{aligned}
\Phi^*\left(w\right) &= \max_{\mu\geq 0,\ \lambda}\Phi\left(\lambda, \mu, w\right) \\
&= \max_{\mu\geq 0,\ \lambda}\left(\min_{p_i(t),\ u_i(t-1)} L\left(\lambda, \mu, p_i(t), u_i(t-1), w\right)\right)
\end{aligned} \tag{5}
$$

Because of w, equation (5) is not decomposable, and the subgradient of equation (4) is difficult to obtain. However, it is possible to define a surrogate subgradient presented in Zhao et al. 1999 as a proper direction for updating the multipliers. Define the following subproblems for unit i,

$$
\min_{p_i(t), u_i(t-1)} L_i, \tag{6}
$$

where

$$
\begin{aligned}
L_i = &\sum_{t=1}^{T}\left[C_i\left(p_i(t)\right)+S_i\left(x_i(t), u_i(t)\right)\right. \\
&\left.-\lambda(t)p_i(t)-\mu(t)r_i\left(x_i(t), p_i(t)\right)\right] \\
&+w\left(\sum_{t=1}^{T}\left|d\_val\_d(t)-p_i(t)\ \right|\right) \\
&+w\left(\sum_{t=1}^{T}\left|\min\left\{0, r_i(t)+d\_val\_r(t)\right\}\right|\right)
\end{aligned}
$$

and

$$
d\_val\_d(t)=P_d(t)-\sum_{\substack{j=1 \\ j\neq i}}^{I}p_j(t)
$$

$$d\_val\_r(t)=\sum_{\substack{j=1\\j\neq i}}^{I} r_j\left(x_j(t),p_j(t)\right)-P_r(t)$$

For equation (6), the constraints of single unit must be satisfied, $d\_val\_d(t)$ and $d\_val\_r(t)$ are the extent of violation in systems constraints except current unit. When the ith subproblem is solved, the related variables $d\_val\_d(t)$ and $d\_val\_r(t)$ can be treated as constants. Then, equation (6) can be solved by using the method described in reference (Guan et al. 1992; Zhao et al. 1999). In this case, the solutions of the identical subproblems are possibly different since the factors in penalty, $d\_val\_d(t)$ and $d\_val\_r(t)$, may have been changed, and so as to change

$$w\left(\sum_{t=1}^{T}\left|d\_val\_d(t)-p_i(t)\ \right|\right)+w\left(\sum_{t=1}^{T}\left|\min\left\{0,r_i(t)+d\_val\_r(t)\right\}\right|\right)$$

and to change the "cost function". Then the problem of identical units is transferred to the problem of different units.

The method named PSS (Penalized Surrogate Subgradient) is summarized as follows:

1. Initialization:

For the given $\lambda^0,\mu^0\ (\mu^0\geq 0)$, $w=0$ is set in (4) and the Standard Lagrangian Relaxation method is used in solving all subproblems to get $p_i^0(t)$, $u_i^0(t)$ (Guan et al. 1992). Then let

$\widetilde{L}=L\left(p_i^0(t),u_i^0(t),0,\mu^0,w^0\right)$ with $w^0>0$. Update λ^0 to

$$\lambda^0(t)=\alpha\left(P_d(t)-\sum_{i=1}^{I}p_i^0(t)\right)$$

where $$\alpha<\frac{\Phi^*(w)-\widetilde{L}}{\sum_{t=1}^{T}\left[P_d(t)-\sum_{i=1}^{I}p_i^0(t)\right]^2+\sum_{t=1}^{T}\left[P_r(t)-\sum_{i=1}^{I}r_i\left(x_i^j(t),p_i^j(t)\right)\right]^2}$$

From the choice of λ^0, we have $L^0=L\left(p_i^0(t),u_i^0(t),\lambda^0,\mu^0,w\right)<\Phi^*(w)$, which makes $L^0<\Phi^*(\lambda^*,\mu^*,w^0)$.

2. Updating multipliers:

Define the components of surrogate subgradients (Wang et al. 1995) as follows:

$$g_\lambda^j=\left[g_\lambda^j(1),g_\lambda^j(2),\cdots,g_\lambda^j(T)\right]^T,$$

$$g_\mu^j=\left[g_\mu^j(1),g_\mu^j(2),\cdots,g_\mu^j(T)\right]^T,$$

and $g_\lambda^j(t)=P_d(t)-\sum_{i=1}^{I}p_i^j(t)$,

$$g_\mu^j(t)=P_r(t)-\sum_{i=1}^{I}r_i\left(x_i^j(t),p_i^j(t)\right).$$

(t = 1, 2, …, T).

Then update the multipliers:

$\lambda^{j+1}(t) = \lambda^{j}(t) + s^{j} g_{\lambda}^{j}(t)$,

$\mu^{j+1}(t) = \mu^{j}(t) + s^{j} g_{\mu}^{j}(t)$.

where s^j is the step size at step j and satisfies

$$0 < s^j < \frac{\Phi^*(w) - L^j}{\left\|g_\lambda^j\right\|^2 + \left\|g_\mu^j\right\|^2},$$

$$L^j = L\left(\lambda^j, \mu^j, p_i^j(t), u_i^j(t), w^0\right).$$

3. Solving subproblems:

Subproblems are solved sequentially to find $p_i^{j+1}(t)$ and $u_i^{j+1}(t)$ until they can satisfy

$$\begin{aligned} L^{j+1} &= L\left(p_i^{j+1}(t), u_i^{j+1}(t), \lambda^{j+1}, \mu^{j+1}, w^0\right) \\ &\le L\left(p_i^{j}(t), u_i^{j}(t), \lambda^{j+1}, \mu^{j+1}, w^0\right) \end{aligned}$$, then go to next step.

4. Checking stopping criteria

The stopping criteria can be the number of loops, or the difference between the solutions of two loops, or the mode of surrogate subgradient. If the criteria are satisfied, stop looping. If not, go back to step 3.

4. Implementation and Testing Result

This PSS method is implemented by C++ and run on a computer configured with PIII 800MHz, 512M RAM. A case of a short-term scheduling with 10 units is given. The data and units' parameters are in appendix. In this case, No. 3-5 units and No. 6-8 units are identical. The scheduling period is 24 hours. The results are compared with Standard Lagrangian Relaxation method (SLR). With the final solution, we can draw a conclusion that PSS is better than SLR.

Table1 The compare of dual gap between two methods

	Dual Gap	Dual Cost ($)	Feasible Cost ($)	Violation Degree
SLR	0.0606433	600907.22	639700.79	4720.2
PSS	0.0128019	601768.68	608699.76	716.35

Table 1 shows the dual gap, dual cost, feasible cost and violation degree. The feasible solution can be evaluated by dual gap. The smaller the dual gap is, the better this feasible solution is. We can find that in Table 1, the feasible cost of PSS is less than that of SLR, and the dual gap and violation degree of PSS are smaller than that of SLR. So it can be seen from table 1 that the PSS is better than SLR.

Table 2 The dual solution of two methods

	SLR	PSS		
HOUR	UNIT3-5	UNIT3	UNIT4	UNIT5
1	0	0	0	0
2	0	0	0	0
3	0	25	0	0
4	72.95	72.95	25	0
5	93.5	93.5	93.5	0
6	107.2	107.2	107.2	107.2
7	114.05	114.05	114.05	114.05
8	114.05	114.05	114.05	114.05
9	114.05	25	120.9	120.9
10	127.75	25	131.75	127.75
11	134.6	25	161.2	134.6
12	141.45	25	162	149.55

To further illustrate the effect of introducing the factor of penalty, table 2 shows the final dual solution. Due to limited space, only the solutions of hour 1 - 12 are shown. In table 2, the identical units have the same dual solutions; however, the dual solutions are different from the solutions obtained by SLR, because the solution oscillations are avoided. Moreover, it shows that the generation of each unit by using PSS is different and the summary of them in every hour is less than the generation obtained by SLR.

This PSS method can be used in power-scheduling systems for the thermal or hydrothermal scheduling modules, especially which have more same (or similar) units in it.

5. Conclusion

Lagrangian Relaxation-based methods have serious and inherent disadvantages in solving unit commitment problems with the identical units. In this paper, a Penalized Surrogate Subgradient method is developed, analyzed and applied to solve unit commitment problems with identical units by introducing a penalty factor and combining the surrogate subgradient method. The commitment of the identical units can be differentiated and homogenous oscillations are avoided. Numerical testing for a short-term generation scheduling problem with several groups of identical units shows the results of the new method are very impressive and the quality of feasible solution is significantly improved. Because of the efficiency of PSS, it can be used in the system which has many identical or similar units in order to get the more accurate solutions.

Appendix

In this case with 10 thermal generating units, the period of scheduling is 24 hours. The detailed units' parameters are in table A.1 and A.2.

Table A.1 Unit Parameter

Unit No.	Min Generation Level	Max Generation Level	Cool Setup Cost	Hot Setup Cost	Min Up Time	Min Down Time	Setup Time	Must Up/Down Time
1	150	455	10000	5000	8	8	13	0
2	150	455	10000	5000	8	8	13	1
3	25	162	1800	900	6	6	10	0
4	25	162	1800	900	6	6	10	0
5	25	162	1800	900	6	6	10	0
6	25	162	1800	900	6	6	10	1
7	25	162	1800	900	6	6	10	1
8	25	162	1800	900	6	6	10	1
9	20	80	340	170	3	3	5	0
10	10	55	60	30	2	1	1	1

Table A.2 System Demand and Spinning Reserve

Time	Demand	Spinning Reserve	Time	Demand	Spinning Time
1	750	75	13	1450	145
2	800	80	14	1350	135
3	900	90	15	1250	125
4	1000	100	16	1100	110
5	1050	105	17	1050	105
6	1150	115	18	1150	115
7	1200	120	19	1250	125
8	1250	125	20	1450	145
9	1350	135	21	1350	135
10	1450	145	22	1150	115
11	1500	150	23	950	95
12	1550	155	24	850	85

References

Baldick, R. (1995). "Generalized unit commitment problem." *IEEE Transactions on Power Systems*, 10(1), 465-475.

Cohen, A. I., and Sherkat, V. R. (1987). "Optimization-based Methods for Operations Scheduling." *Proceedings of the IEEE*, 75(12), 1574-1591.

Ferreira, L. A. F. M., Andersson, T., Imparato, C. F., Miller, T. E., Pang, C. K., Svoboda, A., and Vojdani, A. F. (1989). "Short-term resource scheduling in multi-area hydrothermal power systems." *International Journal of Electrical Power & Energy Systems*, 11(3), 200-12.

Guan, X., Luh, P. B., Yan, H., and Amalfi, J. A. (1992). "An optimization-based method for unit commitment." *International Journal of Electrical Power & Energy Systems*, 14(1), 9-17.

Lai F, G. X. H. (1999). "A perturbation method to reduce solution oscillations of homogeneous unit commitment." *Systems Engineering: Theory Methodology Application*, 8(2), 53-58.

Shaw, J. J. (1995). "A direct method for security-constrained unit commitment." *IEEE Transactions on Power Systems*, 10(3), 1329-42.

Wang, S. J., Shahidehpour, S. M., Kirschen, D. S., Mokhtari, S., and Irisarri, G. D. (1995). "Short-term generation scheduling with transmission and environmental constraints using an augmented Lagrangian relaxation." *IEEE Transactions on Power Systems*, 10(3), 1294-1301.

Zhao, X., Luh, P. B., and Wang, J. (1999). "Surrogate gradient algorithm for Lagrangian relaxation." *Journal of Optimization Theory and Applications*, 100(3), 699-712.